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Superseding AIR1905

Gas Turbine Coaxial Exhaust Flow Noise Prediction

RATIONALE

This document has been determined to contain basic and stable technology which is not dynamic in nature.

STABILIZED NOTICE

This document has been declared "Stabilized" by the SAE A-21 Aircraft Noise Measure and Noise Aviation Emission Modeling Committee and will no longer be subjected to periodic reviews for currency. Users are responsible for verifying references and continued suitability of technical requirements. Newer technology may exist.

SAE ARP 876C provides a method for predicting major elements of the exhaust noise of a jet. It is incomplete in that no recommended method for shock-free coaxial flows is included.

As discussed in the Addendum (Appendix C) to ARP 876C, the lack of a recommended method reflects the difficulties incurred in attempting to correlate satisfactorily data that have become available since the mid 1970s. During that period, three proposed methods were reviewed by the Gas Turbine Propulsion Subcommittee of the SAE A-21 Aircraft Noise Committee, none of which received the necessary level of support to allow publication in ARP 876C. Accordingly this AIR is issued to make available to those wishing to predict turbofan exhaust noise the main candidate procedures thus far considered by the Subcommittee.

No preference is shown by the SAE A-21 Gas Turbine Propulsion Subcommittee for any of the three methods annexed hereto. Work is continuing to develop an acceptable prediction procedure.

CONTENTS

METHOD 1

(ex-Rolls Royce)

7. PREDICTION PROCEDURE FOR COAXIAL JET NOISE:

7.1 Static Conditions: Prediction of coaxial jet noise is by virtue of modification of the single stream prediction. The method is based on coaxial data where the bypass (fan) flow is cold and the core stream between 600 and 900 degrees Kelvin. Limited tests with both coplanar and extended core nozzles up to 2 1/2 diameters downstream of the bypass nozzle showed little effect.

- The correction to the single stream method is by virtue of a correction Δ, where:

$$\Delta = SPL_{coaxial} - SPL_{primary} \quad \ldots (1)$$

$$= F_1\left\{\frac{V_c}{A_o}, \frac{fD_c}{V_c}, AR, VR, \theta_i\right\} \quad \ldots (2)$$

for given T_c and T_b (core and bypass jet temperatures)

where AR = Area Ratio (A_b/A_c)
VR = Velocity Ratio (V_b/V_c)

From V_c and V_b (core and bypass jet velocities)
D_c and D_b (core and bypass nozzle diameters)
and θ_i = Angle to intake axis.

- Δ_m is defined as the mean value of Δ over the range of primary velocities $650 \leqslant V_c \leqslant 1250$ (ft/sec.)

Then,

$$\Delta_m = F_2\left\{\frac{fD_c}{V_c}, AR, VR, \theta_i\right\} \quad \ldots (3)$$

- The effect of primary velocity on Δ is accounted for in correlations of $\acute{\Delta}$ where

$$\acute{\Delta} = 100\left(\frac{\partial\Delta}{\partial V_c}\right) = F_3\left\{\frac{fD_c}{V_c}, AR, VR, \theta_i\right\} \quad \ldots (4)$$

- The coaxial correction Δ may be calculated as follows:

$$\Delta = \Delta_m + \acute{\Delta}\,\frac{V_c - 274}{30.5} \quad \ldots (5)$$

- Plots of Δ_m against the Strouhal number $\left(\frac{fD_c}{V_c}\right)$ are presented in Figs. 1-16 for values of θ_i, AR and VR as defined below:

  θ_i = 60°, 90°, 120°, 150°
  AR = 1, 2, 4, 6
  VR = 0.2, 0.4, 0.6, 0.8, 1.0

- Plots of $\acute{\Delta}$ against the Strouhal number $\left(\frac{fD_c}{V_c}\right)$ are presented in Figs. 17-19 for values of θ_i, AR and VR as defined below:

  θ_i = 60°, 90°, 120°, 150°
  AR = 1, 2, 4, 6
  VR = 1.0 for θ_i = 150°
  VR = 0.6, 0.8, 1.0 for $60° \leqslant \theta_i \leqslant 120°$

- Where $\acute{\Delta}$ is not defined, it may be assumed zero.

● The method of calculation is as follows:

Step 1

Determine the following:

Primary nozzle diameter D_c ~ m
Secondary nozzle diameter D_b ~ m
Primary jet velocity V_c ~ m/s
Secondary jet velocity V_b ~ m/s
Primary jet temperature T_c ~ °K

Then:

Area ratio AR = (A_b/A_c)

and

Velocity ratio VR = (V_b/V_c)

Step 2

Using Appendix A for single stream jets obtain a plot of the Sound Pressure Level for the primary jet alone at (r, θ_i) in the farfield against 1/3 octave Strouhal frequencies $\left(\frac{fD_c}{V_c}\right)$.

Step 3

Using Figs. 1-16 obtain the 1/3 octave spectrum of Δ_m corresponding to the required angle, area ratio and velocity ratio.

Step 4

Using Figs. 17-19 obtain the 1/3 octave spectrum of $\acute{\Delta}$ corresponding to the required angle, area ratio and velocity ratio.

Step 5

Compute the spectrum of the coaxial correction Δ using Δ_m and $\acute{\Delta}$ from Steps 3 and 4 at corresponding Strouhal frequencies as follows:

$$\Delta = \Delta_m + \acute{\Delta}\left\{\frac{V_c - 274}{30.5}\right\}$$

Step 6

Compute the coaxial jet spectrum by adding the coaxial corrections Δ to the primary jet Sound Pressure Levels of Step 2 at corresponding Strouhal frequencies as follows:

$$SPL_{coaxial} = SPL_{primary} + \Delta$$

Step 7

Obtain the 1/3 octave spectrum levels at position (r, θ_i) and over the range of frequencies from the spectrum of Step 6.

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 1 $\theta_j = 30^\circ$, $\theta_i = 150^\circ$

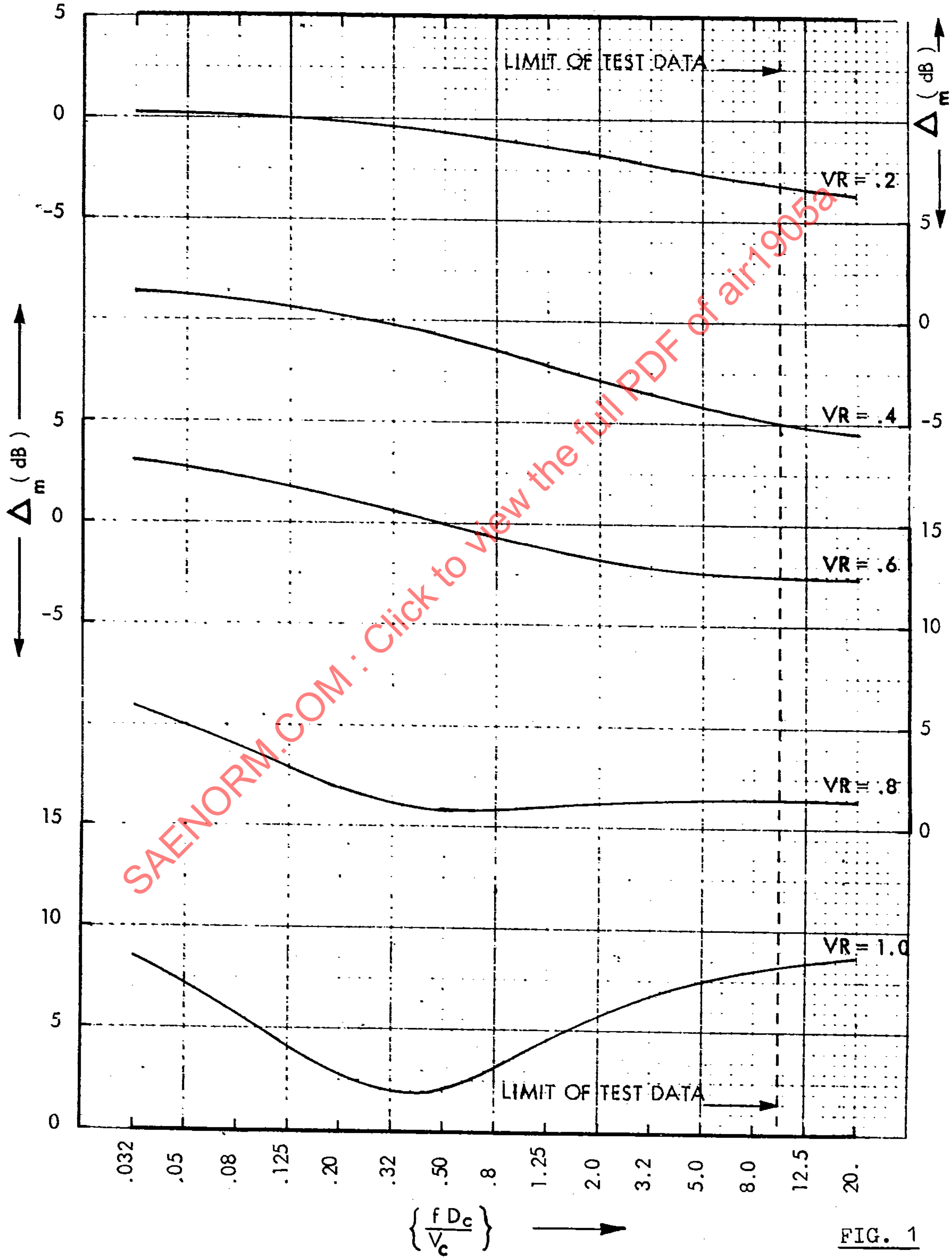

FIG. 1

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 1.0 $\Theta_j = 120^\circ$, $\theta_i = 60^\circ$

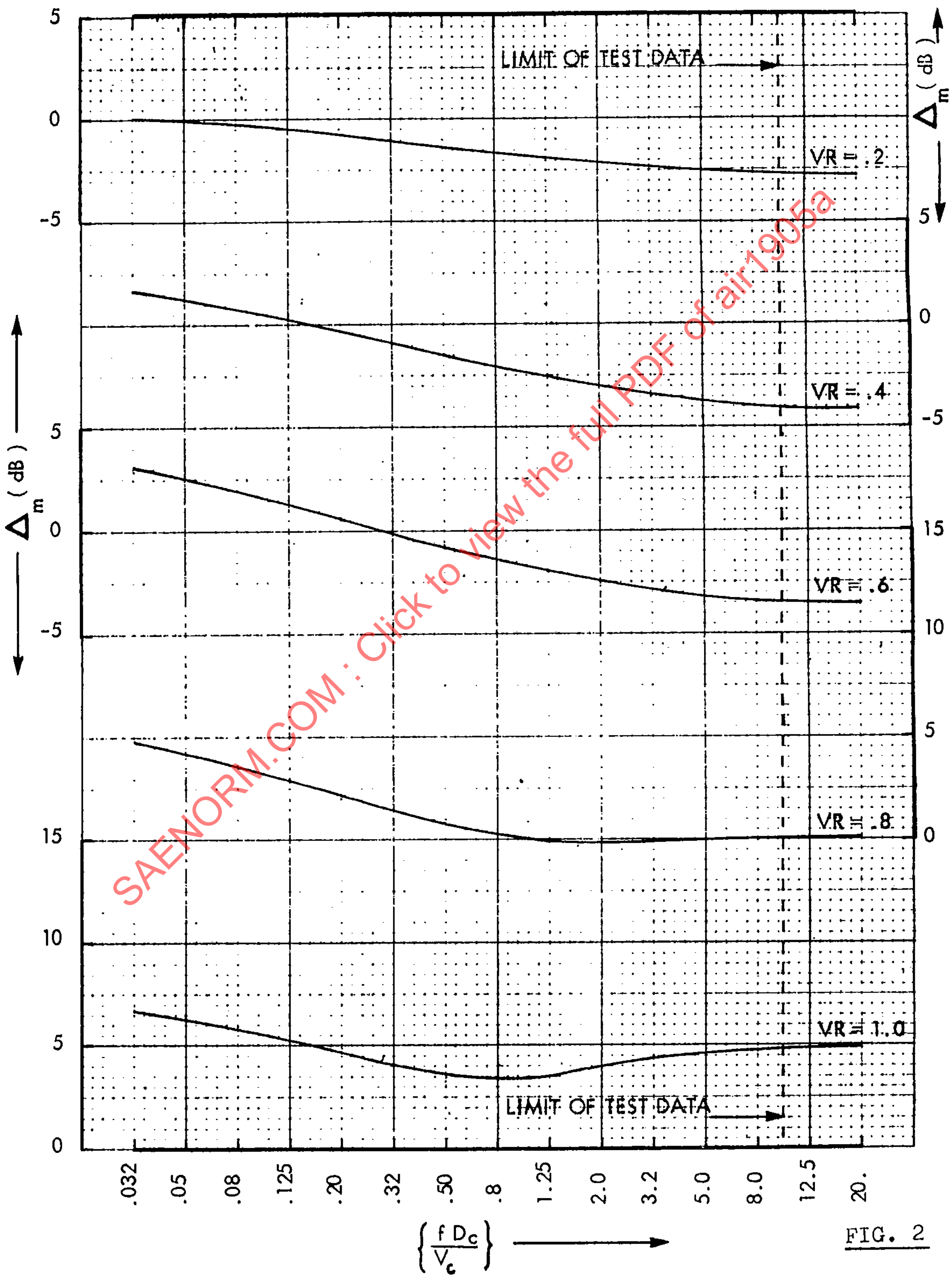

FIG. 2

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 1 $\Theta_j = 90°, \theta_i = 90°$

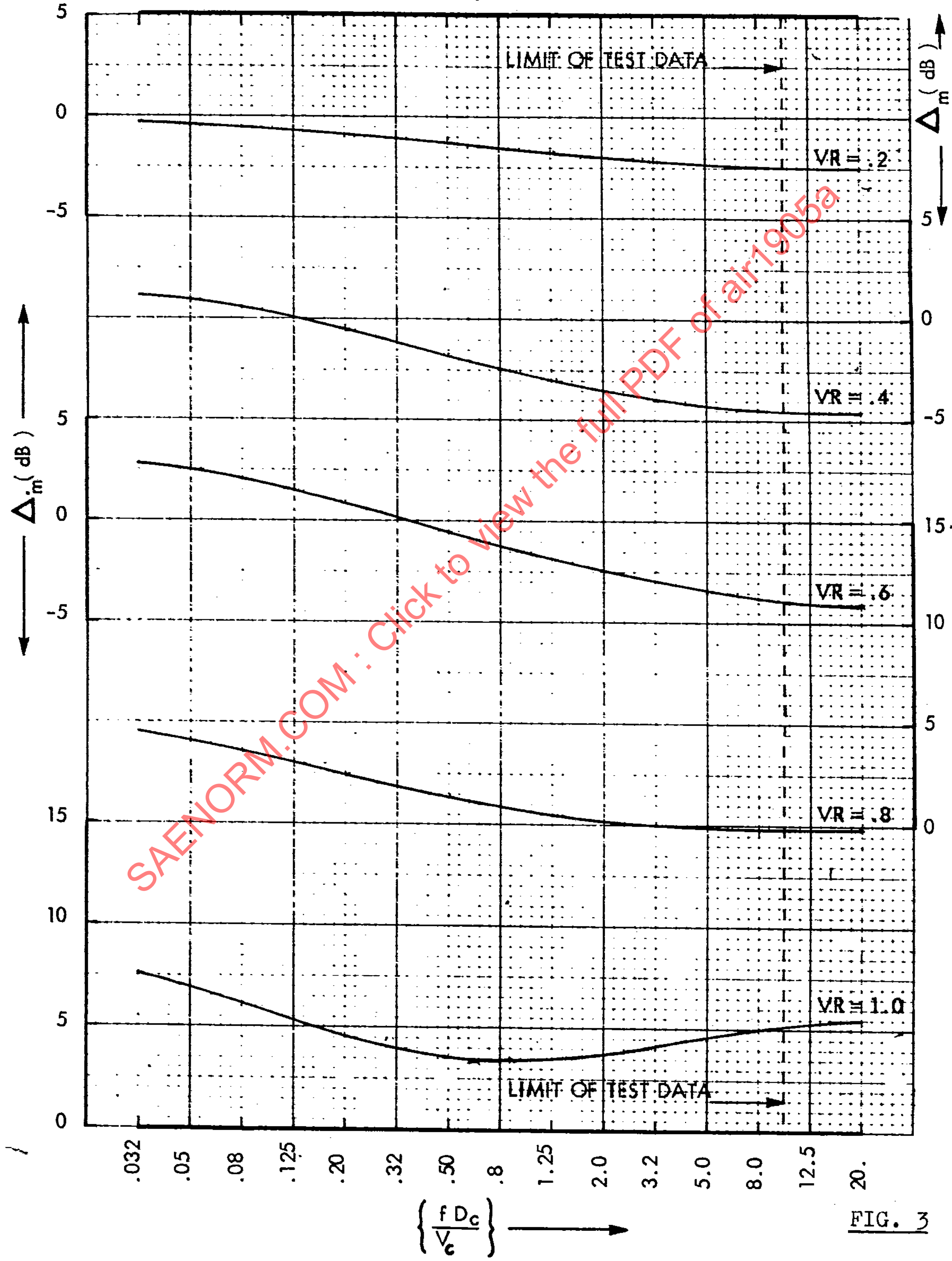

FIG. 3

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 1.0 Θ_j = 60°, θ_i = 120

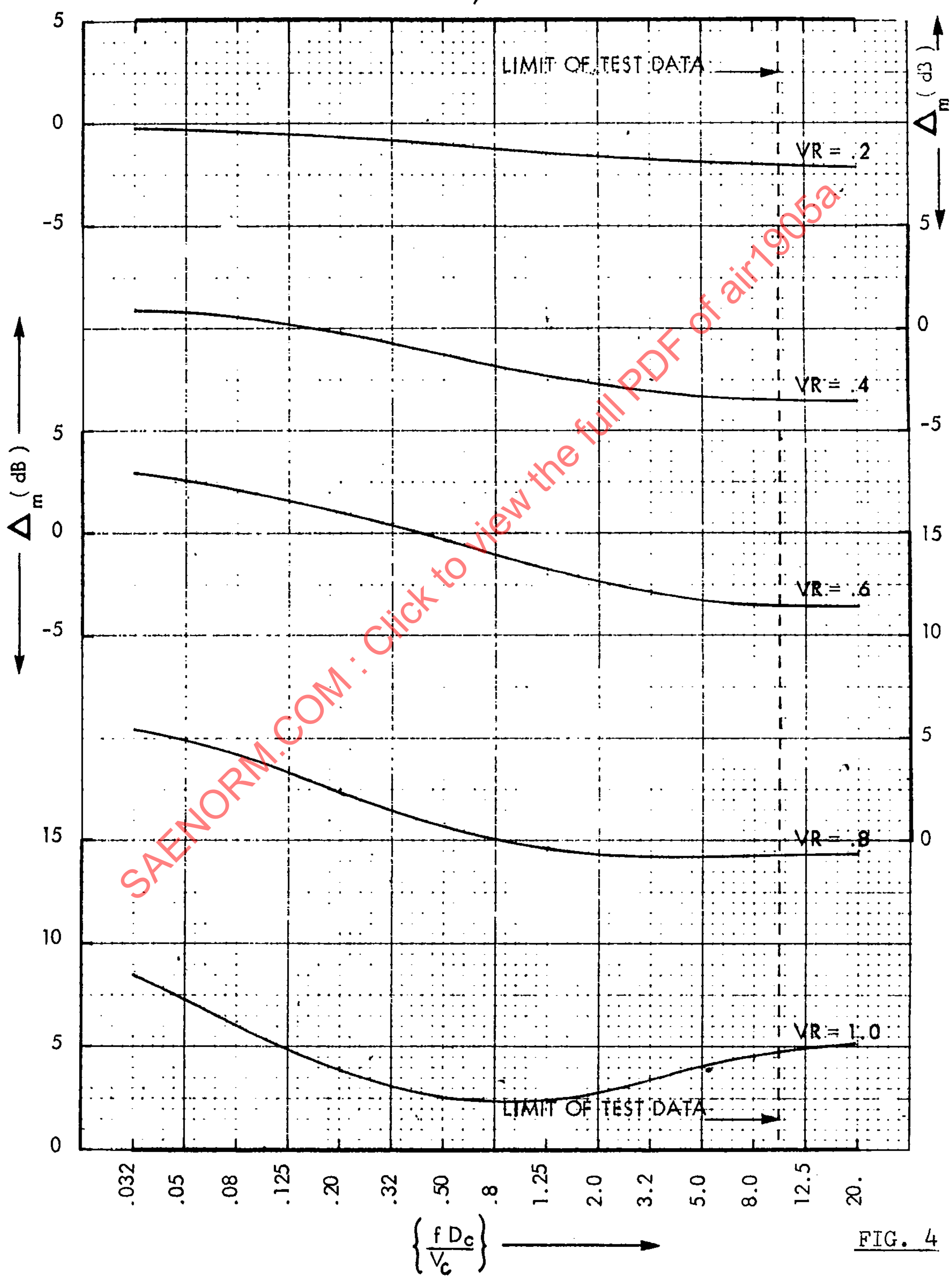

FIG. 4

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 2.0, Θ_j = 30°, θ_i = 150°

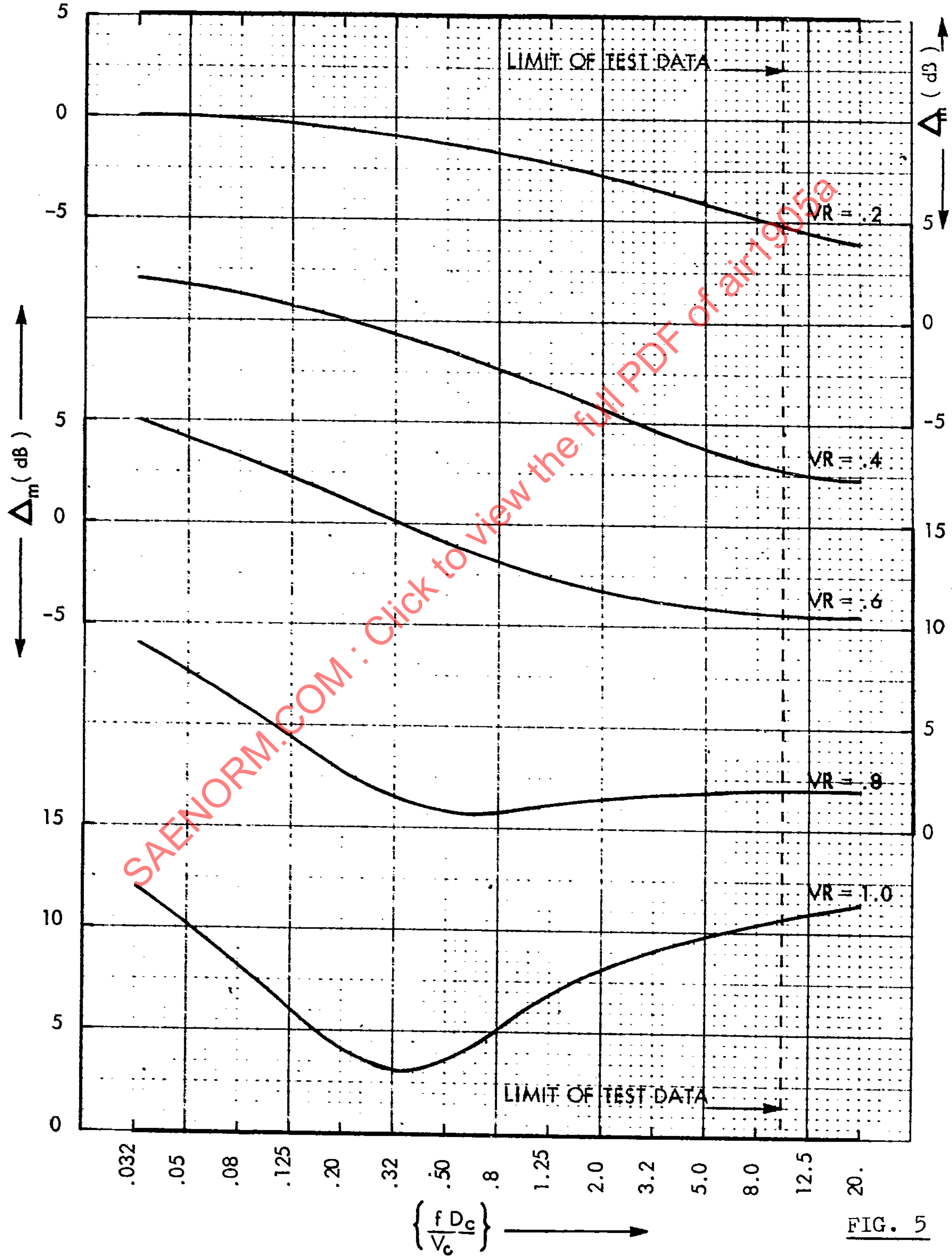

FIG. 5

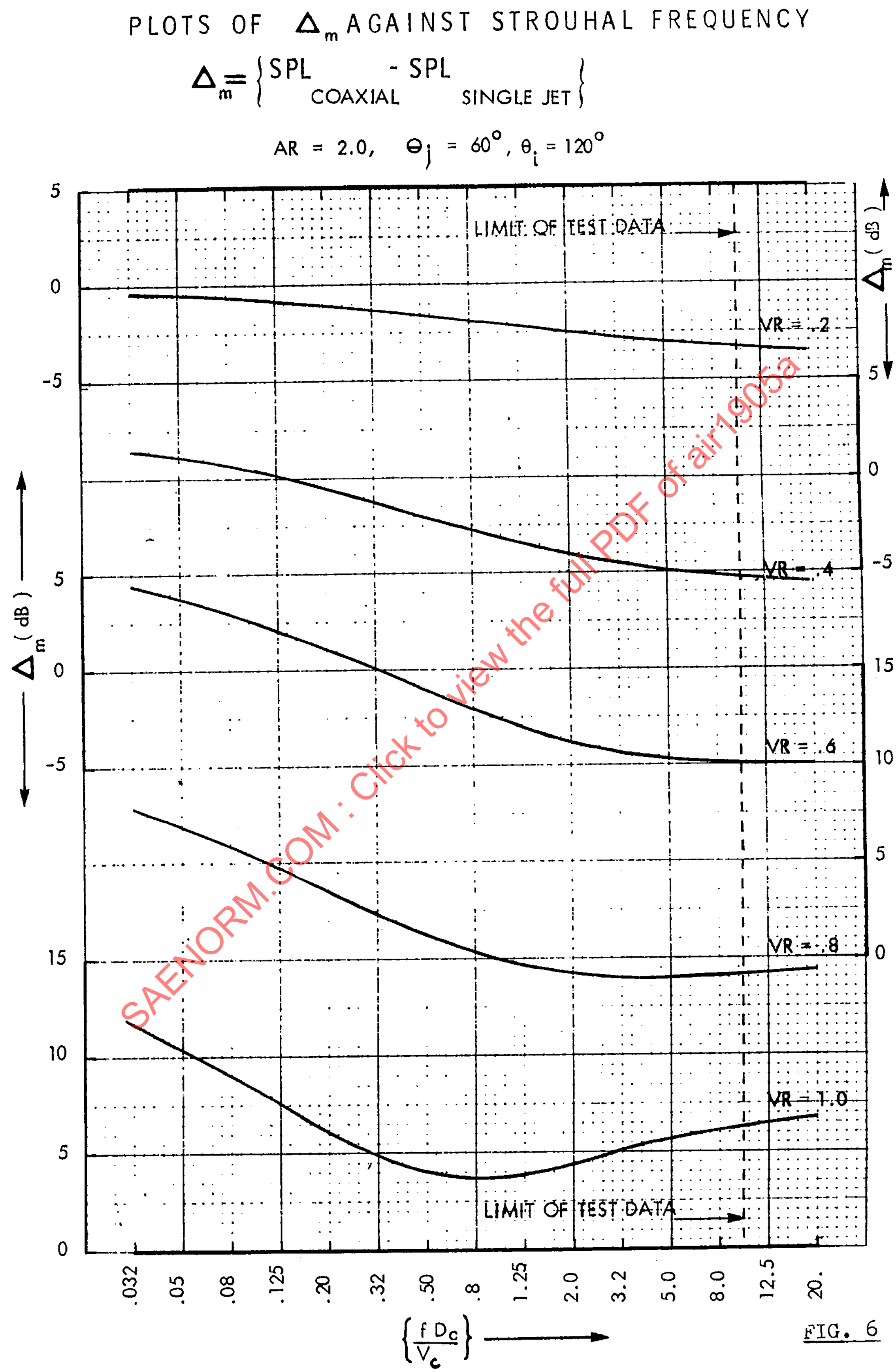
PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY
$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$
AR = 2.0, $\Theta_j = 60°$, $\theta_i = 120°$
LIMIT OF TEST DATA
VR = .2
VR = .4
VR = .6
VR = .8
VR = 1.0
Δ_m (dB)
.032
.05
.08
.125
.20
.32
.50
.8
1.25
2.0
3.2
5.0
8.0
12.5
20.
$\{\frac{f D_c}{V_c}\}$

FIG. 6

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 2 Θ_j = 90°, θ_i = 90°

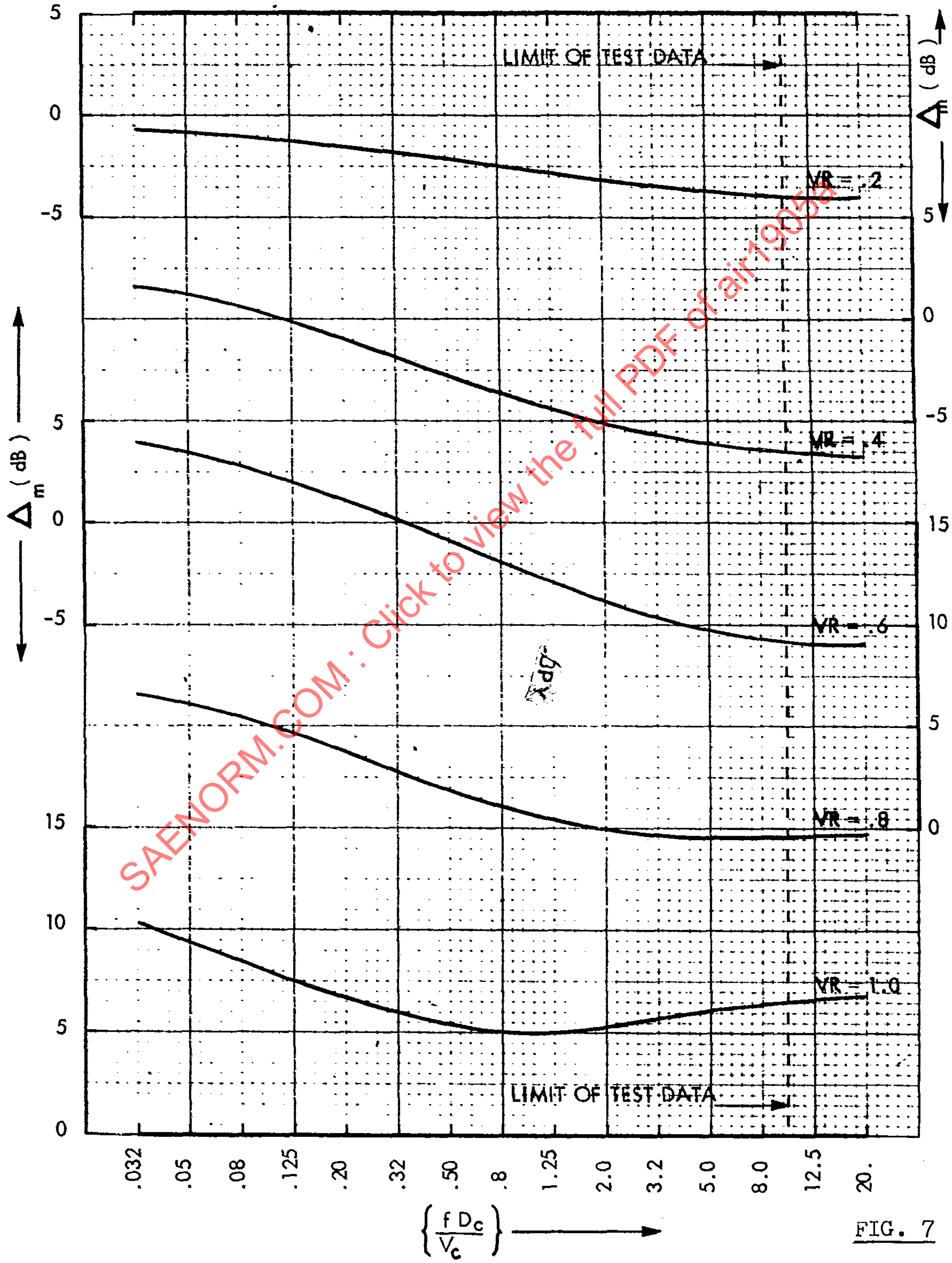

FIG. 7

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 2

$\Theta_j = 120^\circ \quad \theta_i = 60^\circ$

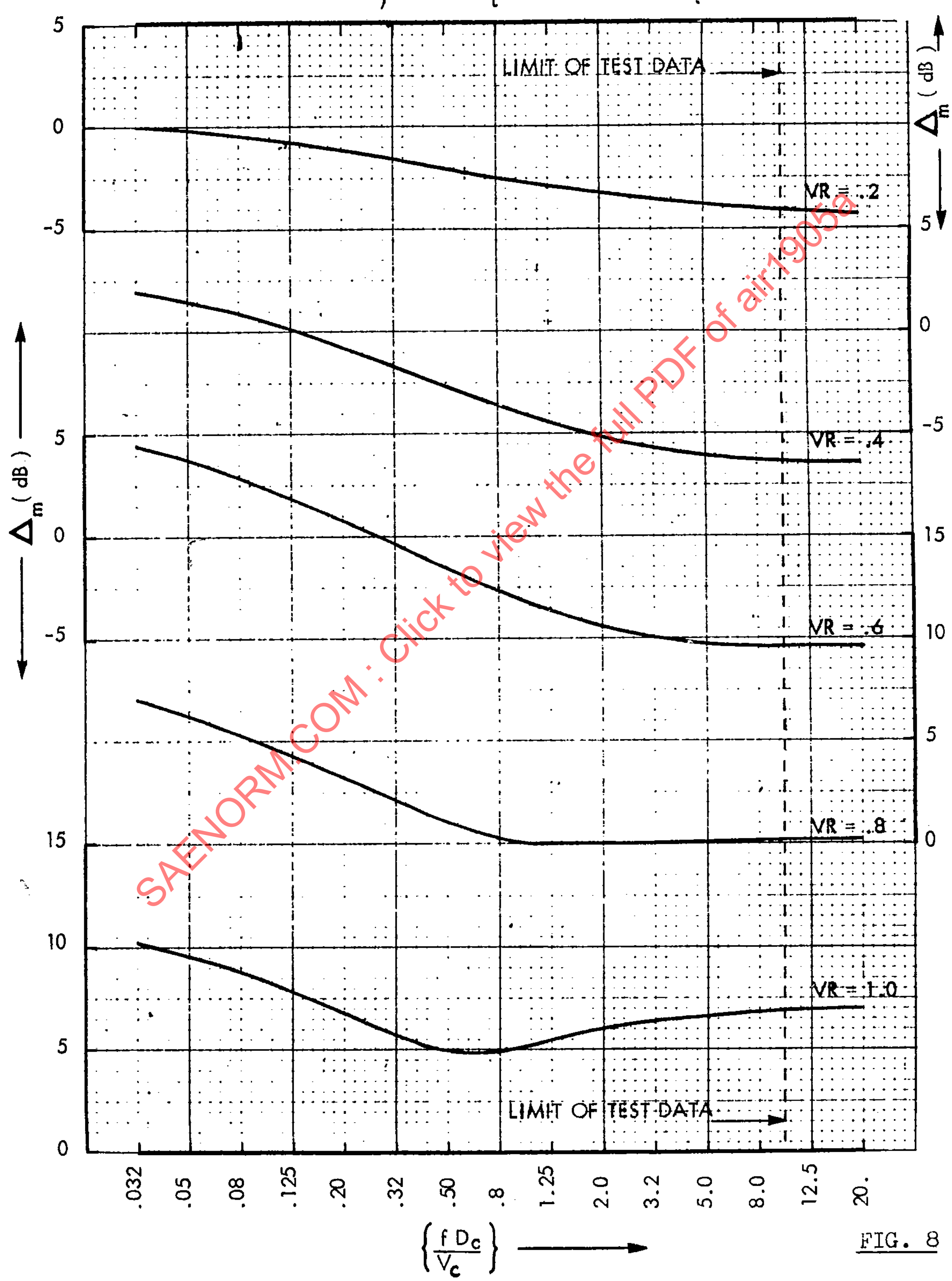

FIG. 8

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 4.0, θ_j = 30°, θ_i = 150°

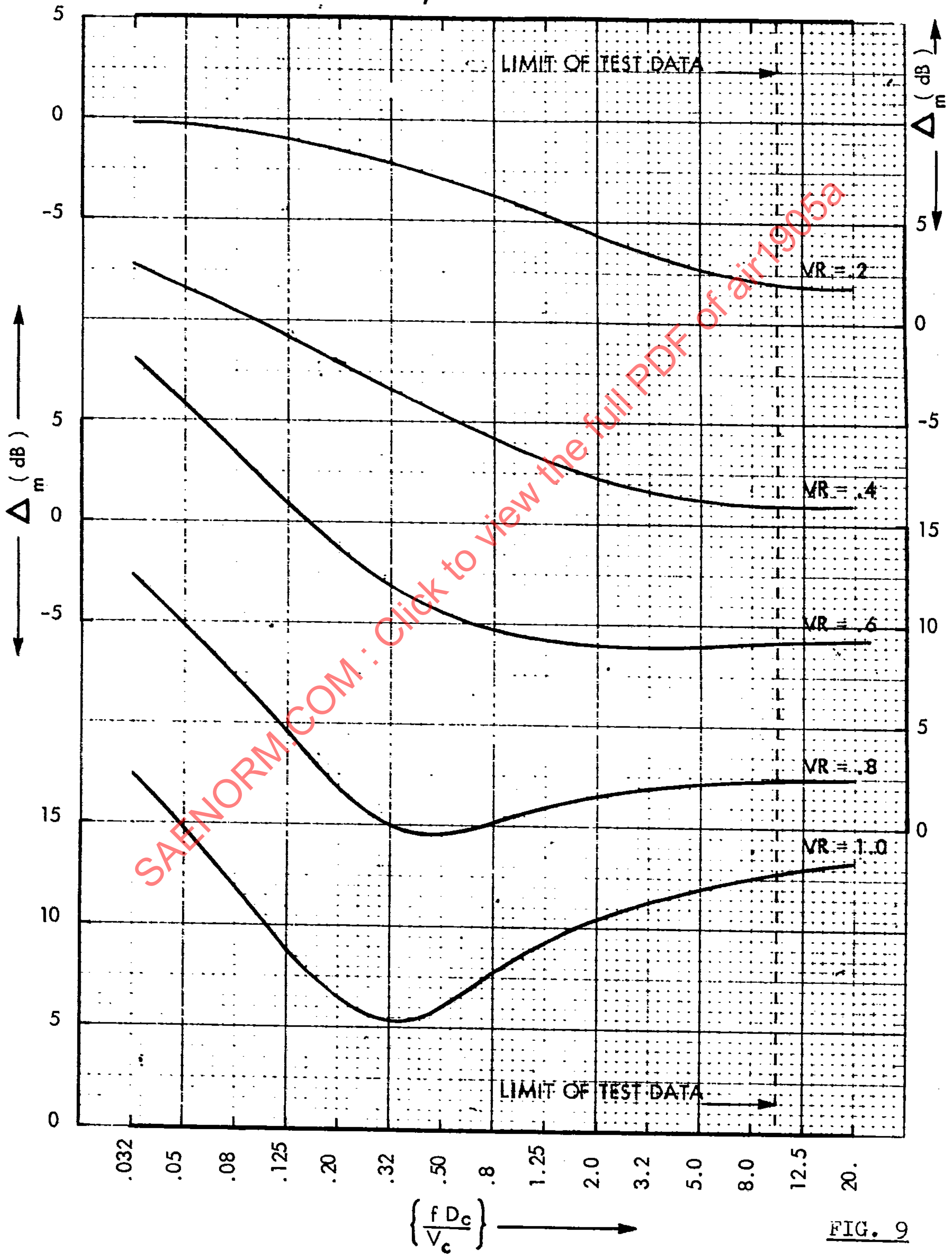

FIG. 9

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 4.0, $\Theta_j = 60^\circ$, $\theta_i = 120^\circ$

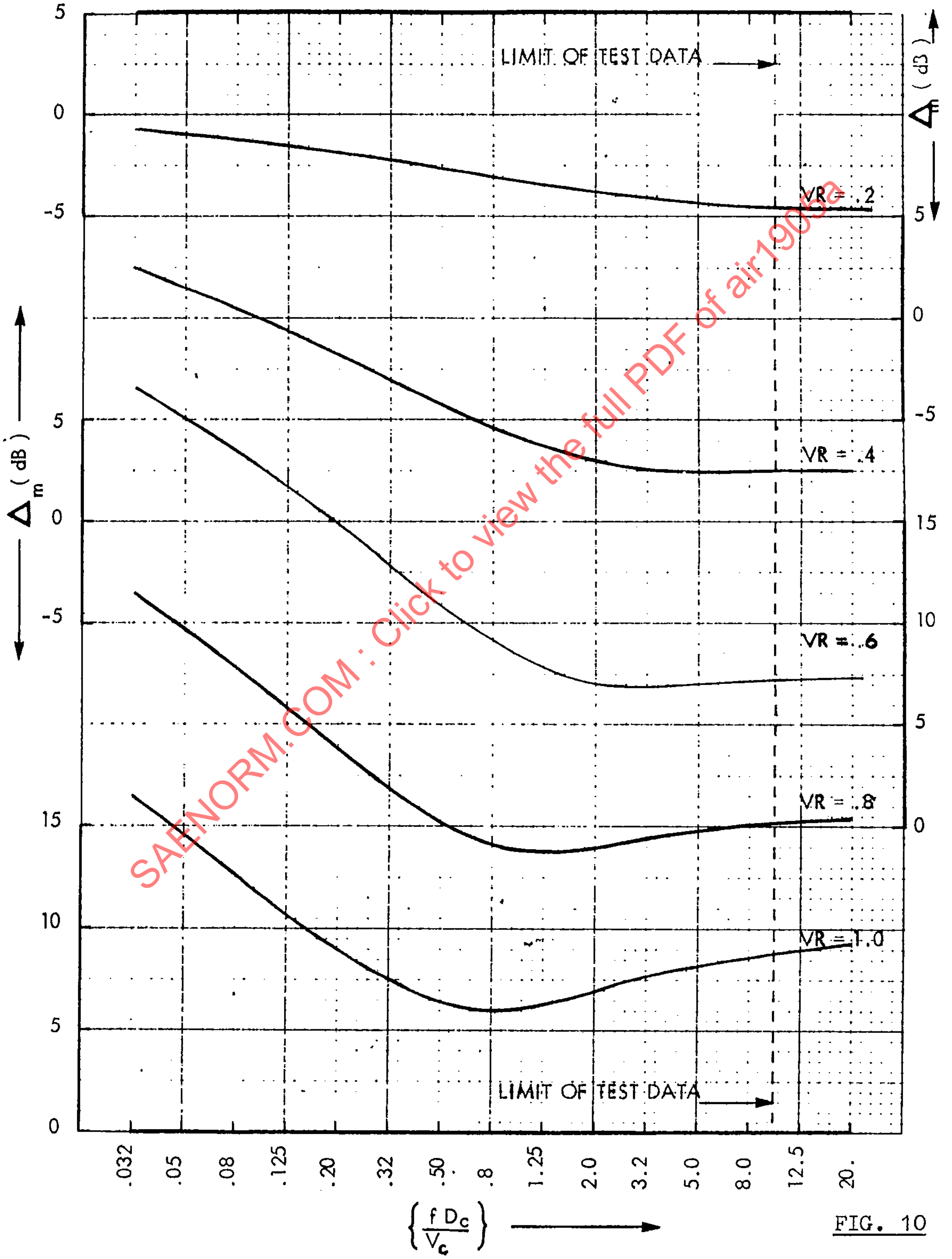

FIG. 10

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 4.0, $\Theta_j = 90^\circ$, $\theta_i = 90^\circ$

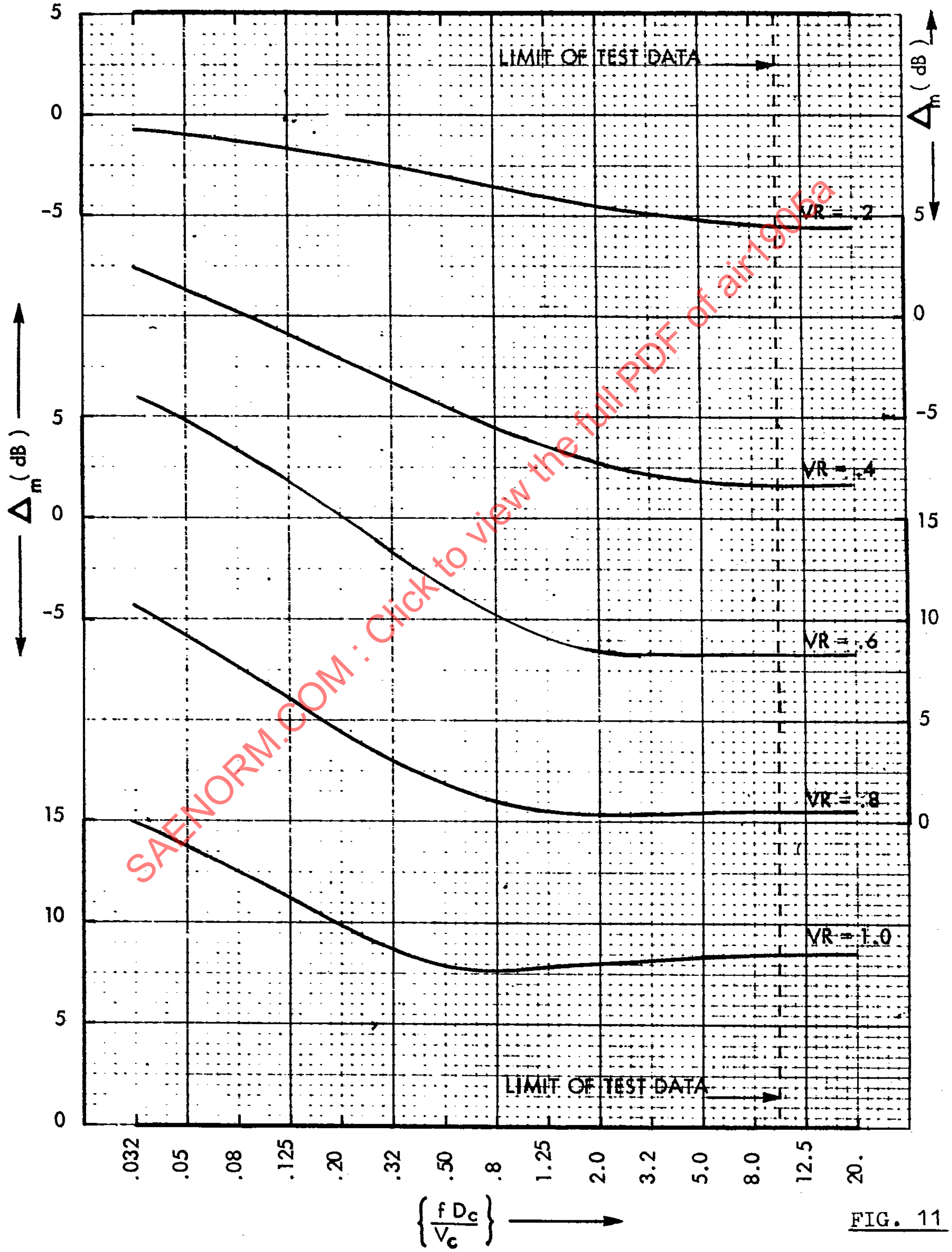

FIG. 11

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 4.0 $\Theta_j = 120^\circ$, $\theta_i = 60^\circ$

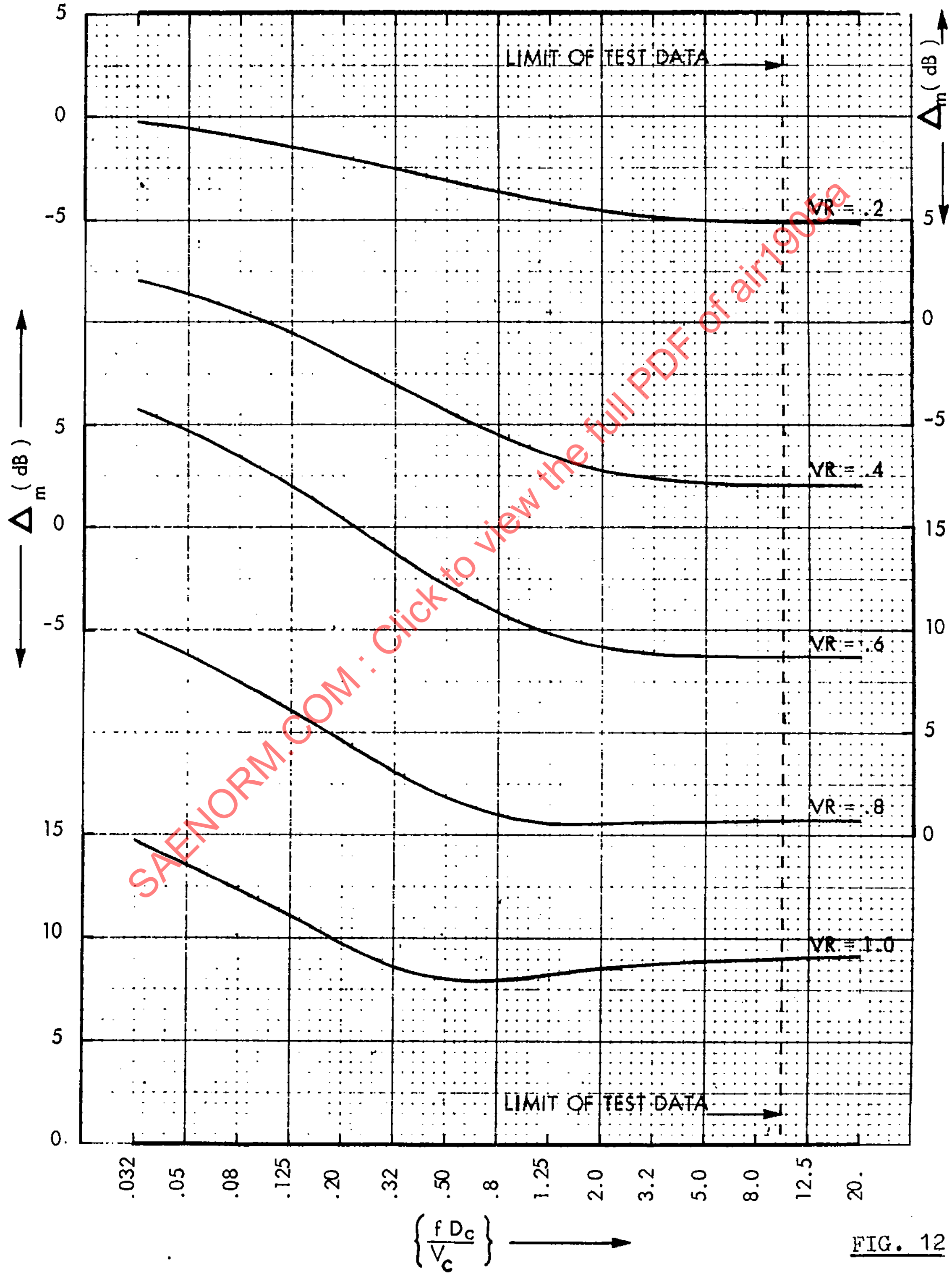

FIG. 12

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 6.0, $\theta_j = 30°$, $\theta_i = 150°$

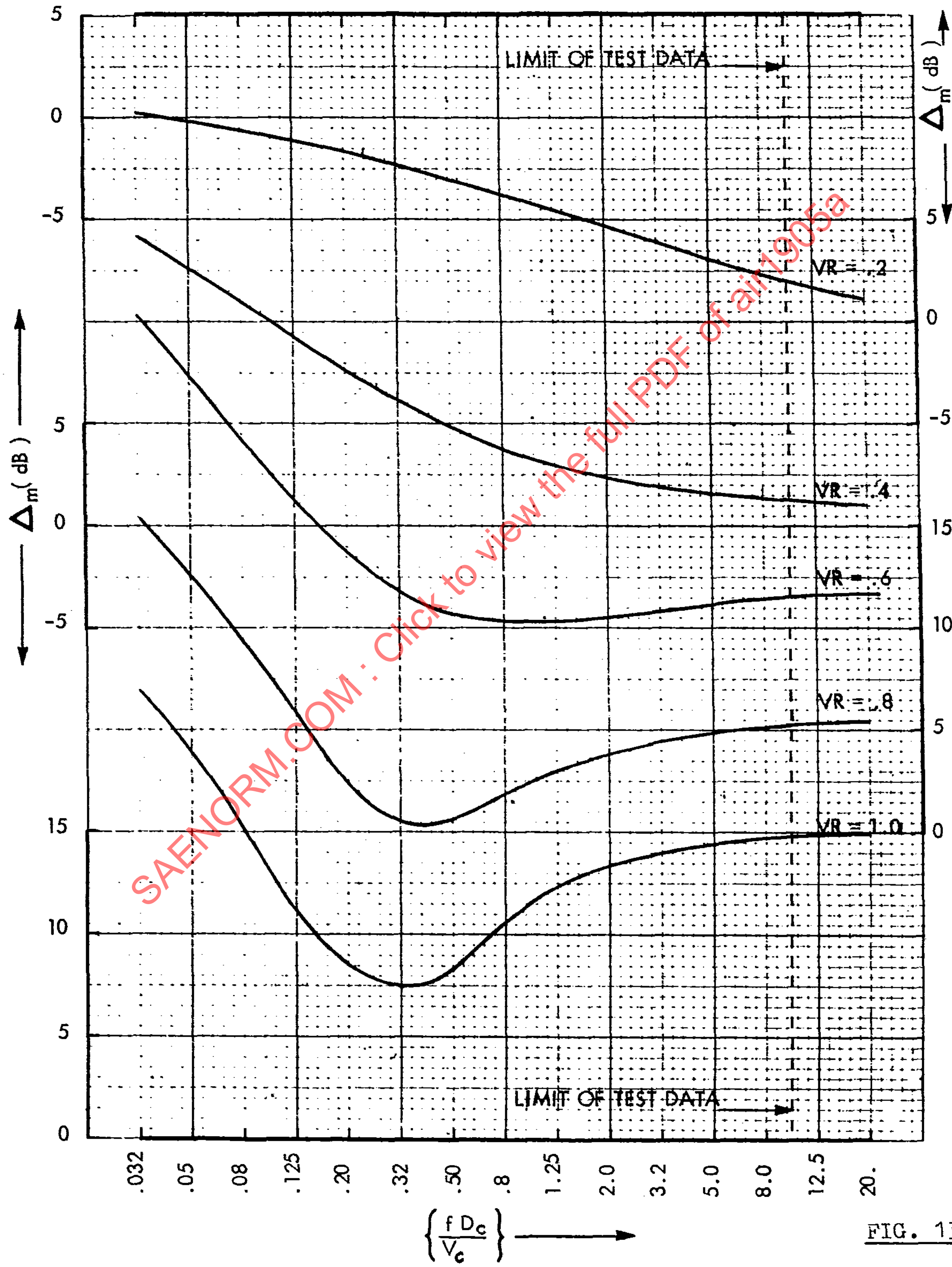

FIG. 13

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \left\{ SPL_{COAXIAL} - SPL_{SINGLE\ JET} \right\}$$

AR = 6.0, $\Theta_j = 60^\circ$, $\theta_i = 120^\circ$

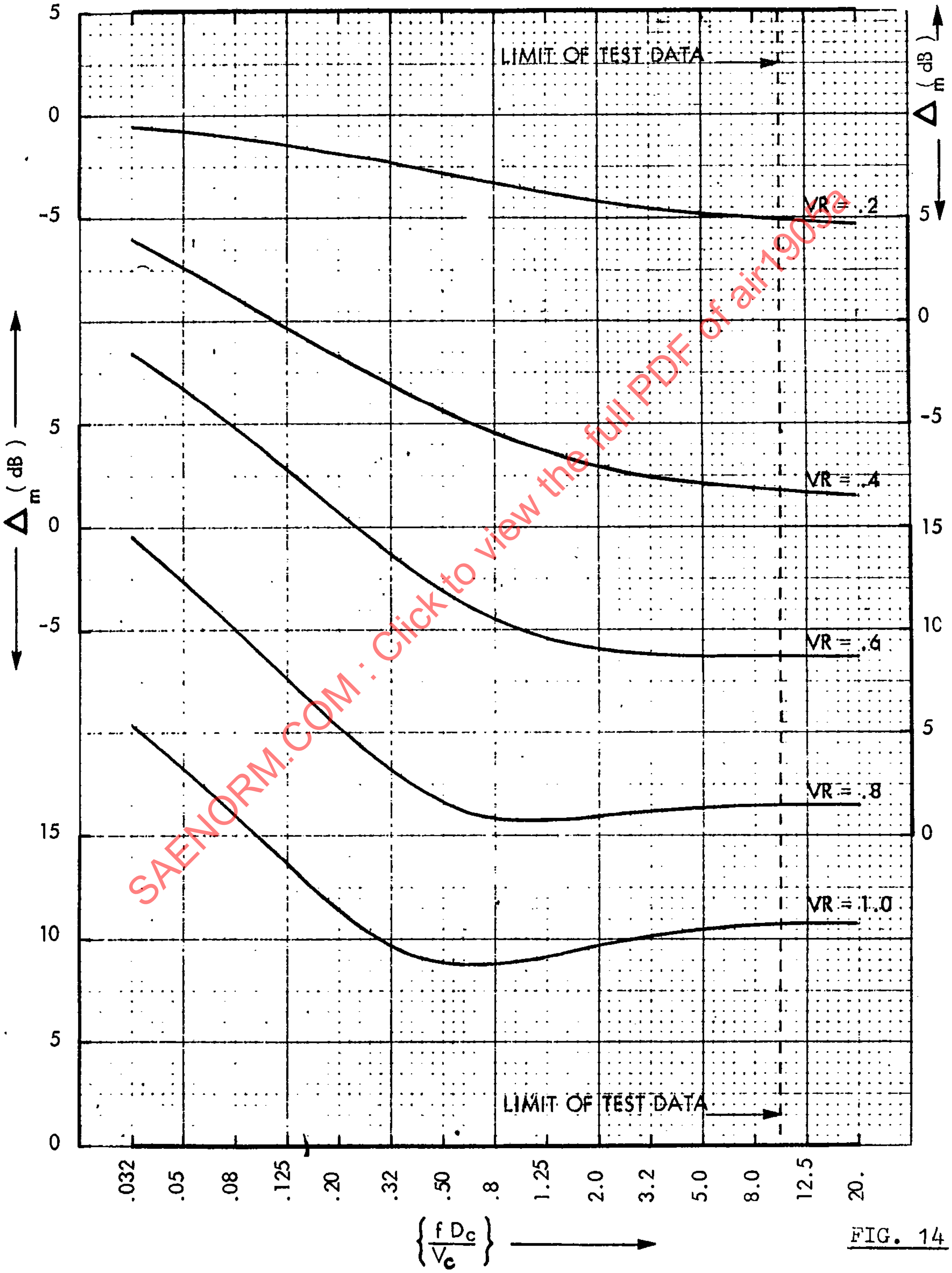

FIG. 14

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 6.0, $\Theta_j = 90°$, $\theta_i = 90°$

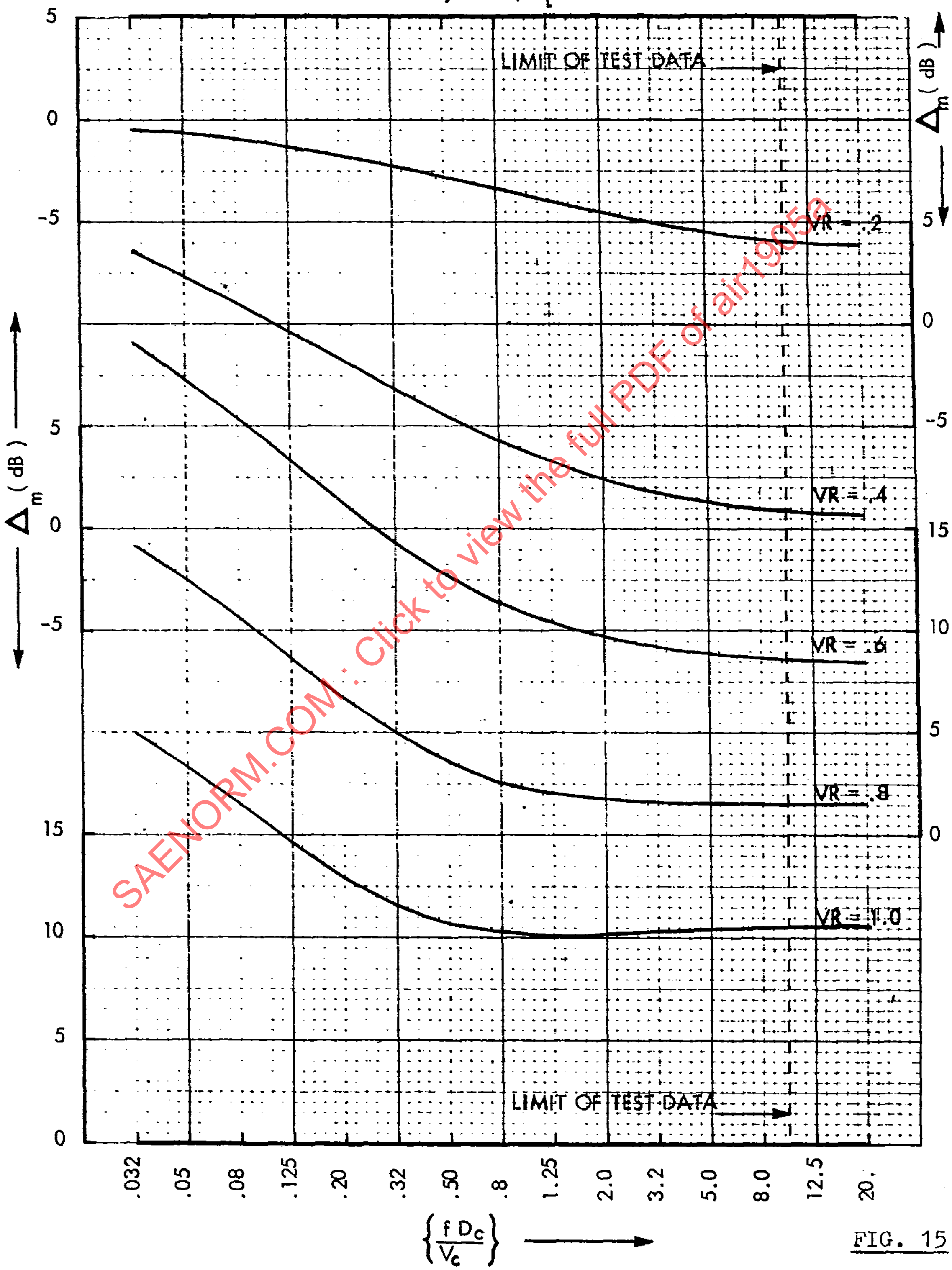

FIG. 15

PLOTS OF Δ_m AGAINST STROUHAL FREQUENCY

$$\Delta_m = \{SPL_{COAXIAL} - SPL_{SINGLE\ JET}\}$$

AR = 6, Θ_j = 120°, θ_i = 60°

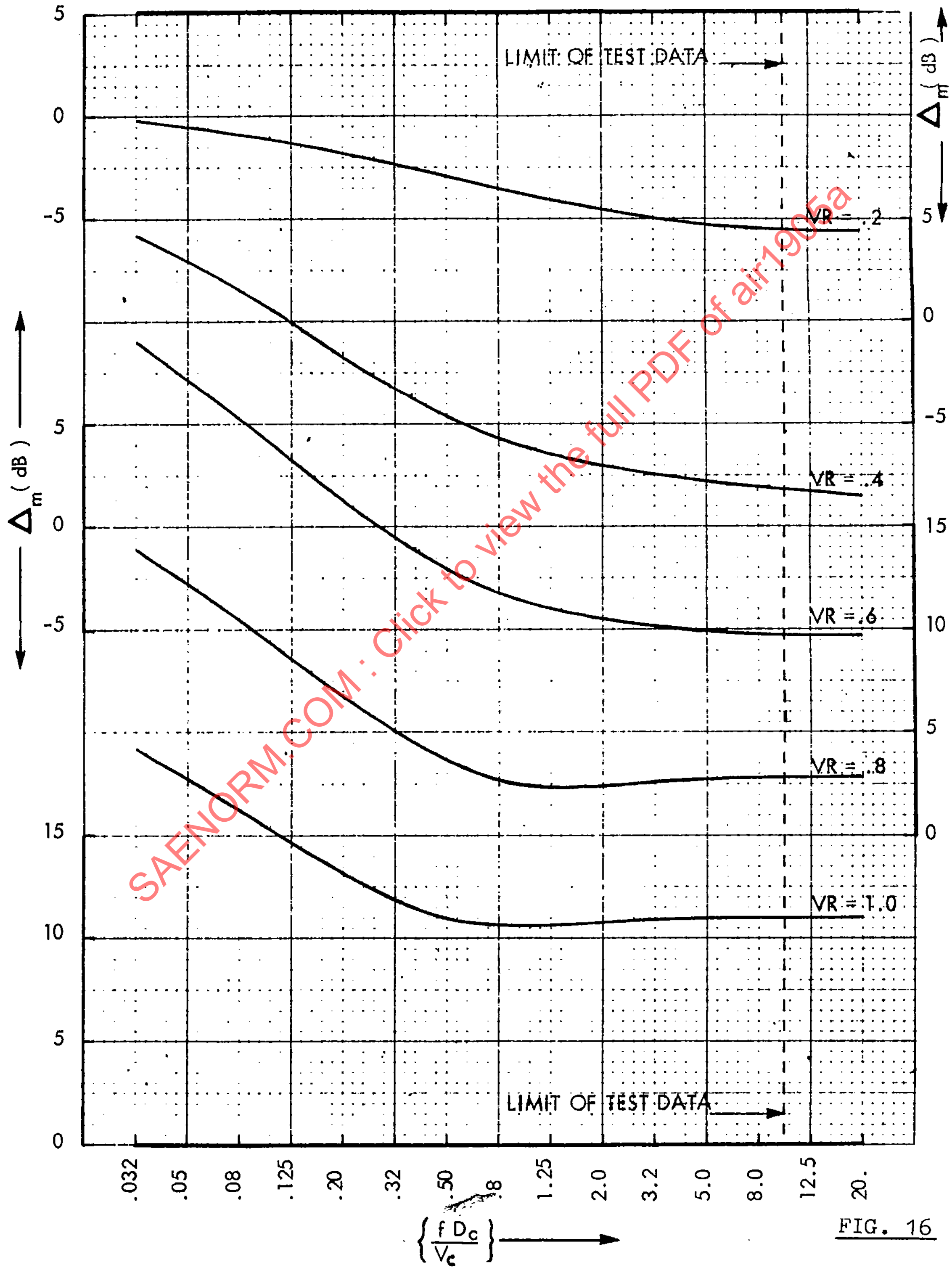

FIG. 16

PLOTS OF Δ' AGAINST STROUHAL FREQUENCY

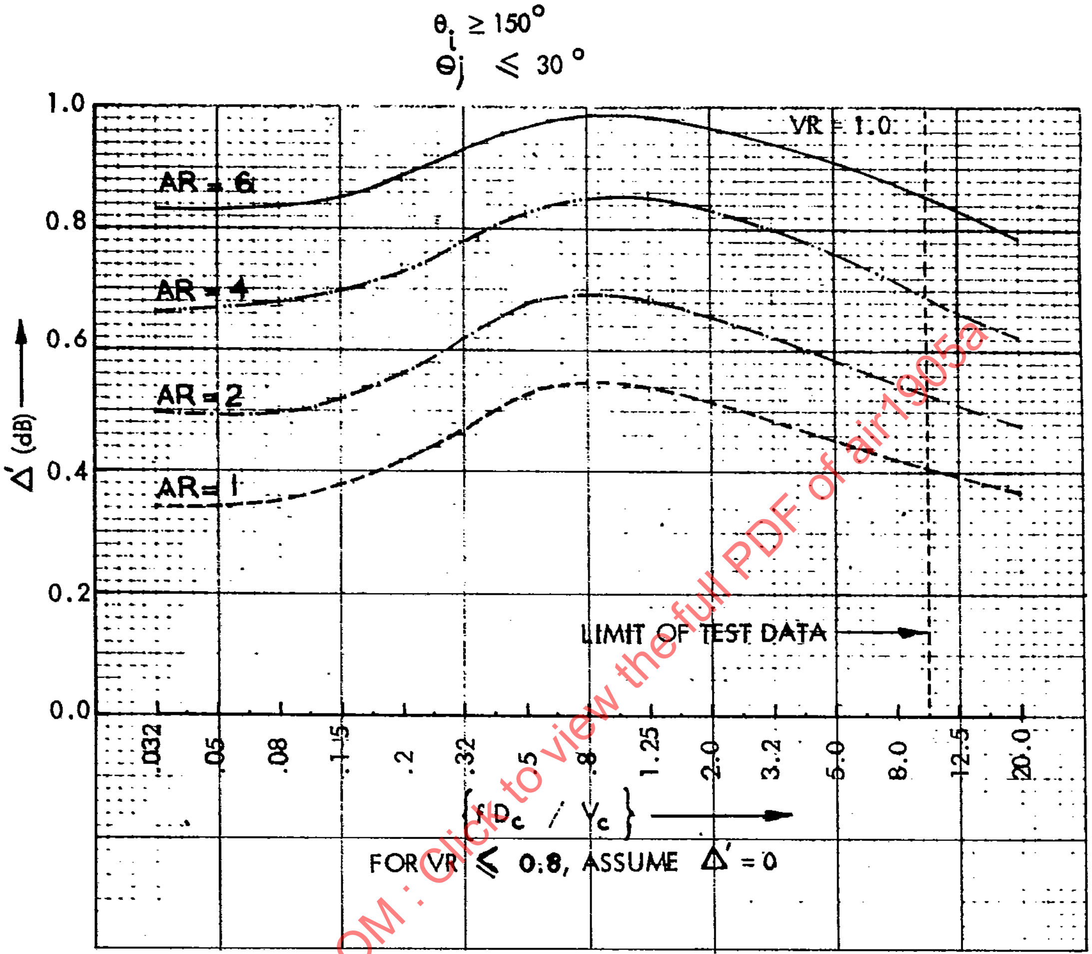

FIG. 17

PLOTS OF Δ' AGAINST STROUHAL FREQUENCY.

$\theta_i = 120^\circ \ \{\theta_j = 60^\circ\}$

——AR=6, —··—··—AR=4, —·—·—AR=2, ------AR=1.

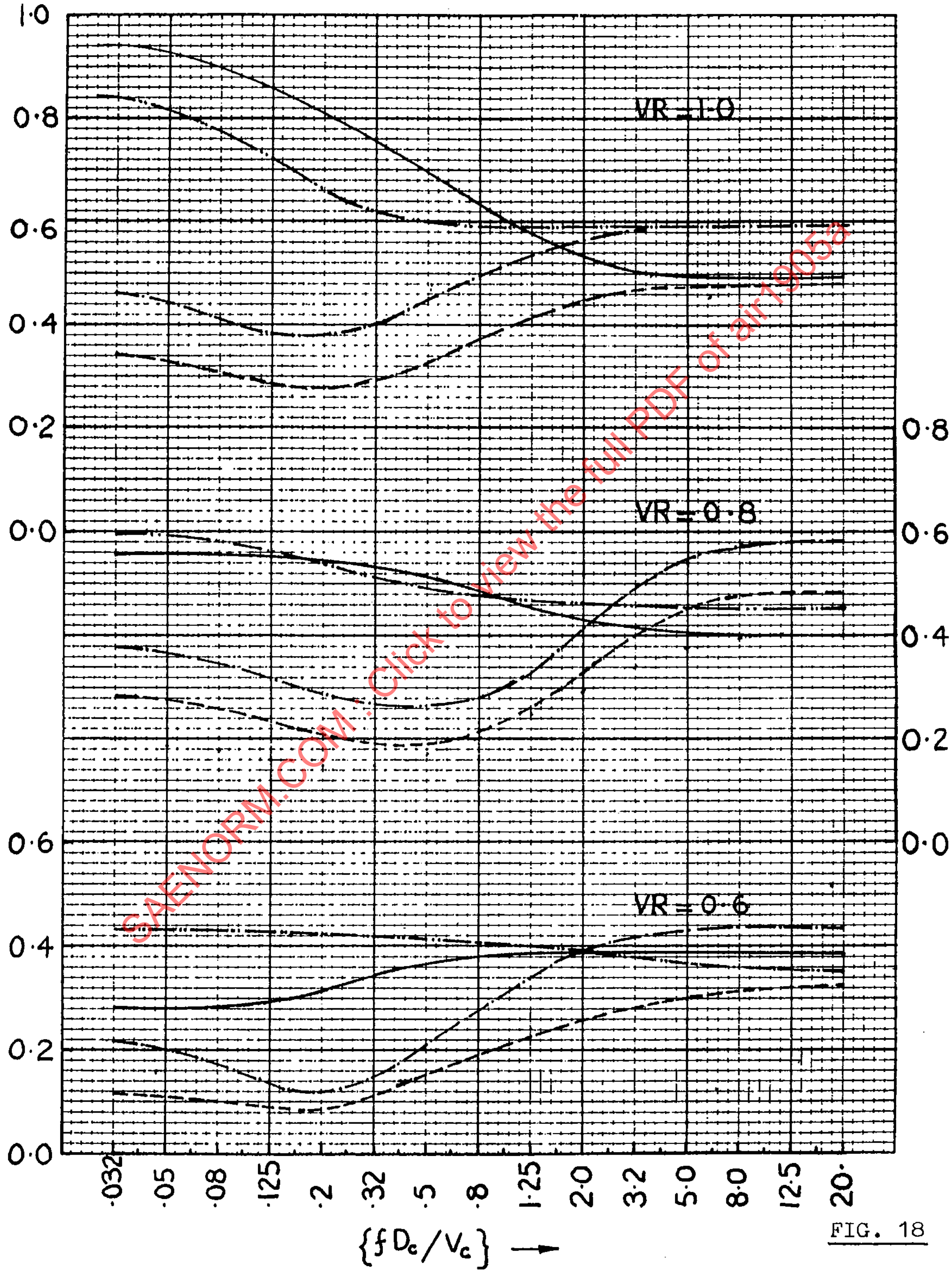

FIG. 18

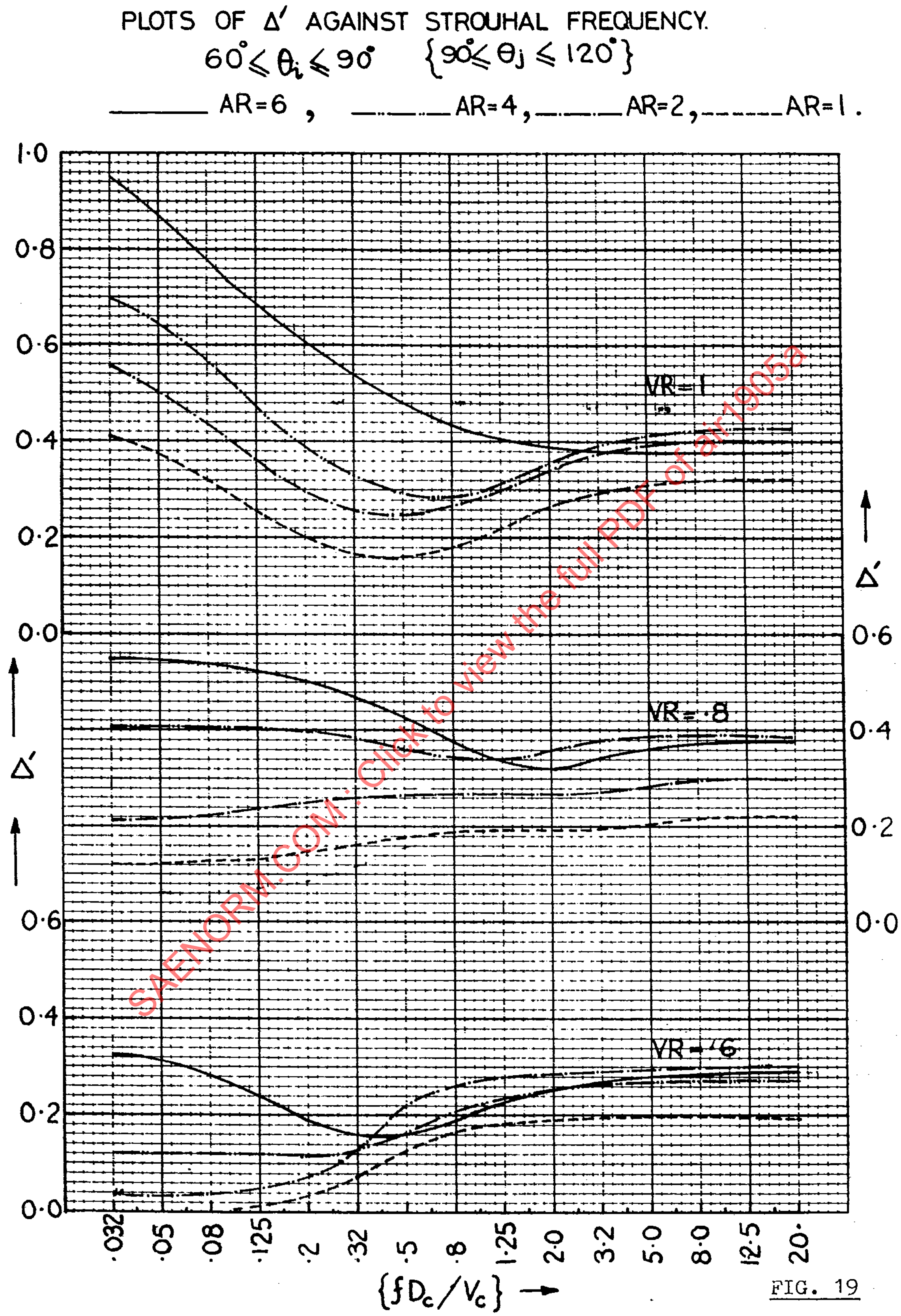
PLOTS OF Δ′ AGAINST STROUHAL FREQUENCY.
60° ≤ θi ≤ 90° {90° ≤ θj ≤ 120°}
AR=6, AR=4, AR=2, AR=1.
VR=1
VR=·8
VR=·6
Δ′
{fDc/Vc}
1·0
0·8
0·6
0·4
0·2
0·0
·032
·05
·08
·125
·2
·32
·5
·8
1·25
2·0
3·2
5·0
8·0
12·5
20·

FIG. 19

METHOD 2

(ex-Boeing)

7. PREDICTION OF SHOCKFREE COAXIAL JET NOISE:

7.1 Static Conditions: Definitive model scale experimental work of recent years has provided a data base for the study of coaxial noise over a wide range of velocity, temperature and area ratios. This work has shown that coaxial jet noise level and Spectral Character is a function of the primary, mixed and secondary jet conditions. The principal parameters of each stream are:

a) The velocity
b) The jet density
c) The jet total temperature
d) The jet dimension and area

It has been concluded that the most convenient way to express jet noise characteristics is to consider firstly the normalized overall sound pressure level (OASPL) as a function of jet velocity (V_j) and angle of measurement (θ_i or θ_j) and to then relate the spectral character (on one-third octave basis) to the overall level at any point in the field. This procedure may be adopted by using Figures B.1 through B.8. The proposed prediction procedure for coaxial jet noise is independent of that for the single jet (Appendix A) except for the density exponent relationship shown in Figure A.1.

The calculation procedure assumes that the following jet flow conditions are available for the primary or inner jet, and the secondary or outer jet;

a. Fully Expanded Mean Jet Velocities, V_p and V_s

b. Fully Expanded Mean Jet Density, ρ_p and ρ_s

c. Mean Total Temperatures, $T_{T,p}$ and $T_{T,s}$

d. Fully Expanded Areas, A_p and A_s

The prediction procedure is restricted to the following range of conditions:

$$1 \leqslant \frac{A_s}{A_p} \leqslant 6$$

$$.4 \leqslant \frac{V_s}{V_p} \leqslant 2.5$$

7.1 (Continued):

$$.33 \leq \frac{T_{T,S}}{T_{T,P}} \leq 3.0$$

$500 \leq V_p \leq 2000$ FPS
$400 \leq V_S \leq 2500$ FPS
$400 \leq V_M \leq 2000$ FPS (see following paragraphs)

The prediction procedure is also limited to configurations without a primary centerbody.

During the data analysis the high velocity data from Reference 1 for an extended primary indicated lower mixed jet OASPL than that for a coplanar configuration, and a strong dependence between peak Strouhal number (Mixed) and mixed jet velocity. These observed differences at high velocities are attributed to an effect of the extended primary nozzle.

Thus, two prediction procedures have been provided:

a. Coplanar and Retracted primary configurations
b. Extended primary configuration

The method of calculation is as follows:

1. Calculate the bypass ratio (BPR)

$$BPR = \frac{\rho_s A_s V_s}{\rho_p A_p V_p}$$

2. Calculate the mixed jet velocity (V_M)

$$V_M = V_p \left\{ \frac{1 + BPR\left(\frac{V_s}{V_p}\right)}{1 + BPR} \right\}$$

3. Calculate the mixed jet total temperature ($T_{T,M}$)

$$T_{T,M} = T_{T,P} \left\{ \frac{1 + BPR \frac{T_{T,S}}{T_{T,P}}}{1 + BPR} \right\}$$

7.1 (Continued):

4. Calculate the mixed jet density (ρ_M)

$$\rho_M = \frac{P_a}{53.3\ T_{s,M}} \qquad \text{where}$$

$$T_{sM} = T_{T,M} - \frac{V_M 2}{2C_{pg} j}$$

5. Calculate the mixed jet area (A_M)

$$A_M = A_p \frac{1 + BPR}{\left(\frac{\rho_M}{\rho_p}\right)\left(\frac{V_M}{V_p}\right)}$$

6. Calculate the mixed jet diameter (D_M)

$$D_M = \left(\frac{4\ A_M}{\pi}\right)^{1/2}$$

7. With V_M, V_S and the ambient speed of sound (a_o) obtain the mixed and secondary jet density exponents (ω_M, ω_S) from Figure A.1.

8. For each desired angle and mixed jet velocity, use Figure B.1 to obtain the free field normalized overall Sound Pressure Level (S), where:

$$S = OASPL - 10\ Log \left(\frac{\rho_M}{\rho_o}\right)^{\omega_M} \left(\frac{A_M}{R^2}\right)\left(\frac{T_{T,M}}{T_o}\right)^{m_1} \left(\frac{T_o}{T_{T,S}}\right)^{m_2}$$

The mixed jet OASPL temperature-directivity exponents n_1 and n_2 at each angle are given in Figure B.1.

9. Calculate the mixed jet OASPL where:

$$OASPL = S + 10\ Log \left(\frac{\rho_M}{\rho_o}\right)^{\omega_M} \left(\frac{A_M}{R^2}\right)\left(\frac{T_{T,M}}{T_o}\right)^{m_1} \left(\frac{T_o}{T_{T,S}}\right)^{m_2}$$

7.1 (Continued):

10. Determine the mixed jet normalized one-third octave band spectral levels using Figure B.2 and

$$\left(\frac{fD_M}{V_M}\right) \frac{1}{\xi_{DF}} \frac{1}{\xi_{MV}}$$

where the Strouhal connection factors ξ_{DF} ξ_{MV} are obtained from Figures B.3 and B.4. Enter Figure B.2 with the corrected mixed jet Strouhal number to determine:

1/3 OBSPL - OASPL

11. Calculate the mixed jet 1/3 octave band sound pressure level,

$$1/3 \text{ OBSPL} = \text{OASPL} + (1/3 \text{ OBSPL} - \text{OASPL})$$

$$= \textcircled{9} + \textcircled{10}$$

12. Correct the mixed jet spectra for the effect of atmospheric attenuation using ARP 866.

$$1/3 \text{ OBSPL}_{CORR} = 1/3 \text{ OBSPL} - \frac{\Delta dB}{1000 \text{ Ft.}} \quad (R-1)$$

13. Determine the normalized secondary jet OASPL using Figure B.5.

$$S = \text{OASPL} - 10 \text{ Log} \left(\frac{\rho_s}{\rho_o}\right)^{\omega_s} \left(\frac{A_s}{R^2}\right) \left(\frac{V_s}{V_p}\right)^{-m_4}$$

14. Calculate the secondary jet OASPL, where

$$\text{OASPL} = S + 10 \text{ Log} \left(\frac{\rho_s}{\rho_o}\right)^{\omega_s} \left(\frac{A_s}{R^2}\right)\left(\frac{V_s}{V_p}\right)^{-m_4}$$

The secondary jet velocity ratio-directivity exponent n_4 is obtained from Figure B.6.

15. Calculate the secondary jet peak Strouhal number $(fD_S/V_S)_{PK}$ for each angle, and secondary velocity using Figure B.7.

7.1 (Continued):

16. Calculate the secondary jet normalized one-third octave band spectral levels using Figure B.8 and

$$\frac{\left(\frac{fD_S}{V_S}\right)}{\left(\frac{fD_S}{V_S}\right)_{PK}} \qquad \left(\frac{V_S}{V_P}\right)^{-n_3}$$

The secondary diameter is that of the outer annulus and is obtained from

$$D_S = \left[\frac{4\,(A_s + A_p)}{\pi}\right]^{1/2}$$

The velocity ratio-Strouhal connection exponent, n_3 is obtained from Figure B.6.

17. The secondary jet 1/3 OBSPL is calculated as follows:

$$1/3\ OBSPL = OASPL + (1/3\ OBSPL - OASPL) = \textcircled{14} + \textcircled{16}$$

18. Correct the secondary jet noise spectra for the effect of atmospheric attenuation using ARP 866.

19. The total coaxial jet noise 1/3 OBSPL is obtained from

$$10\ Log_{10}\left[10^{\frac{SPL_M}{10}} + 10^{\frac{SPL_S}{10}}\right]$$

20. Calculate the OASPL for the coaxial jet as follows:

$$OASPL_{Total} = 10\ Log_{10}\left[\sum_{m=1}^{m=24\ or\ 27} 10^{\frac{SPL_1}{10}} + \ldots + 10^{\frac{SPL_n}{10}}\right]$$

21. Calculate the perceived noise level (PNL) using ARP 865A.

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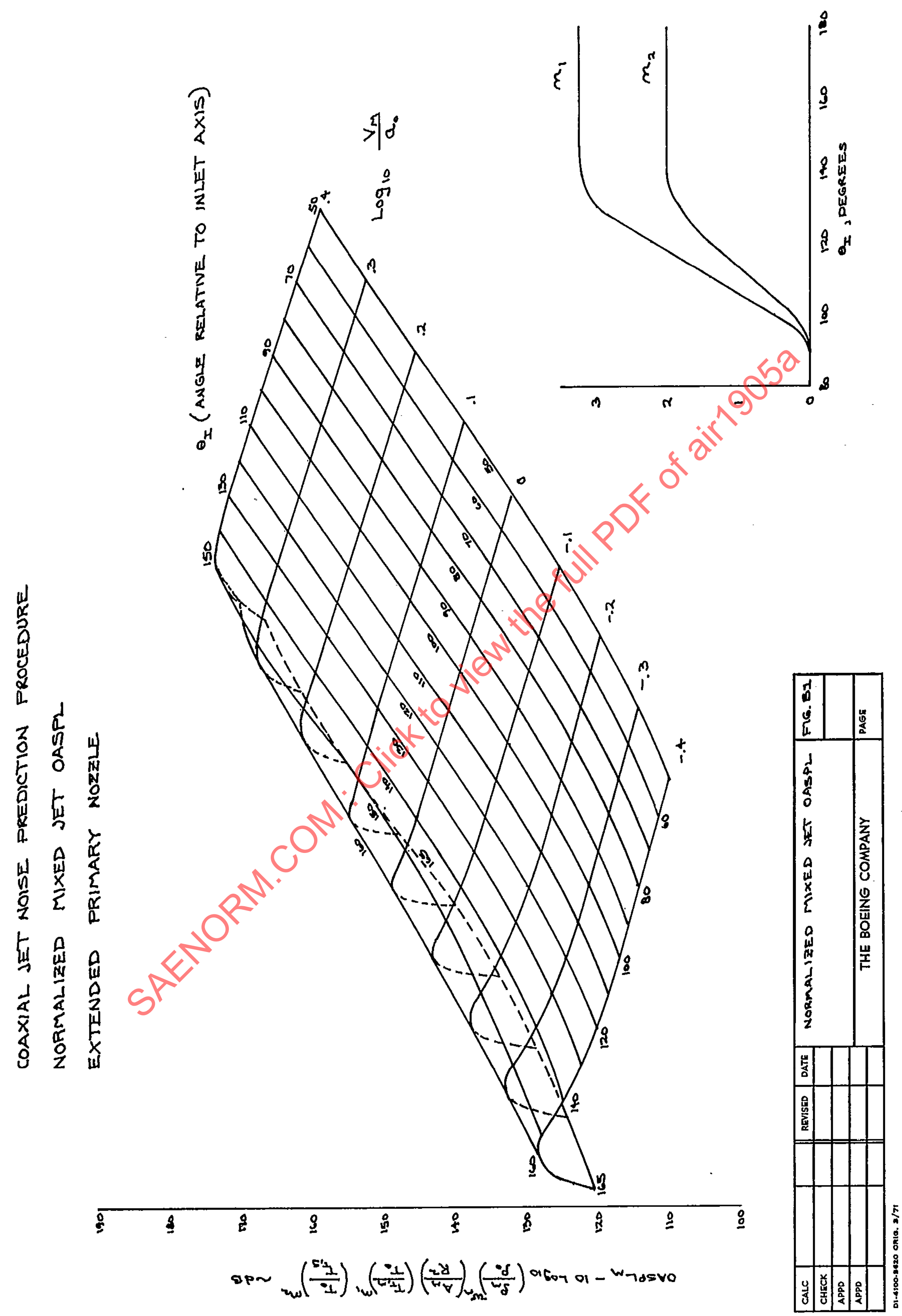
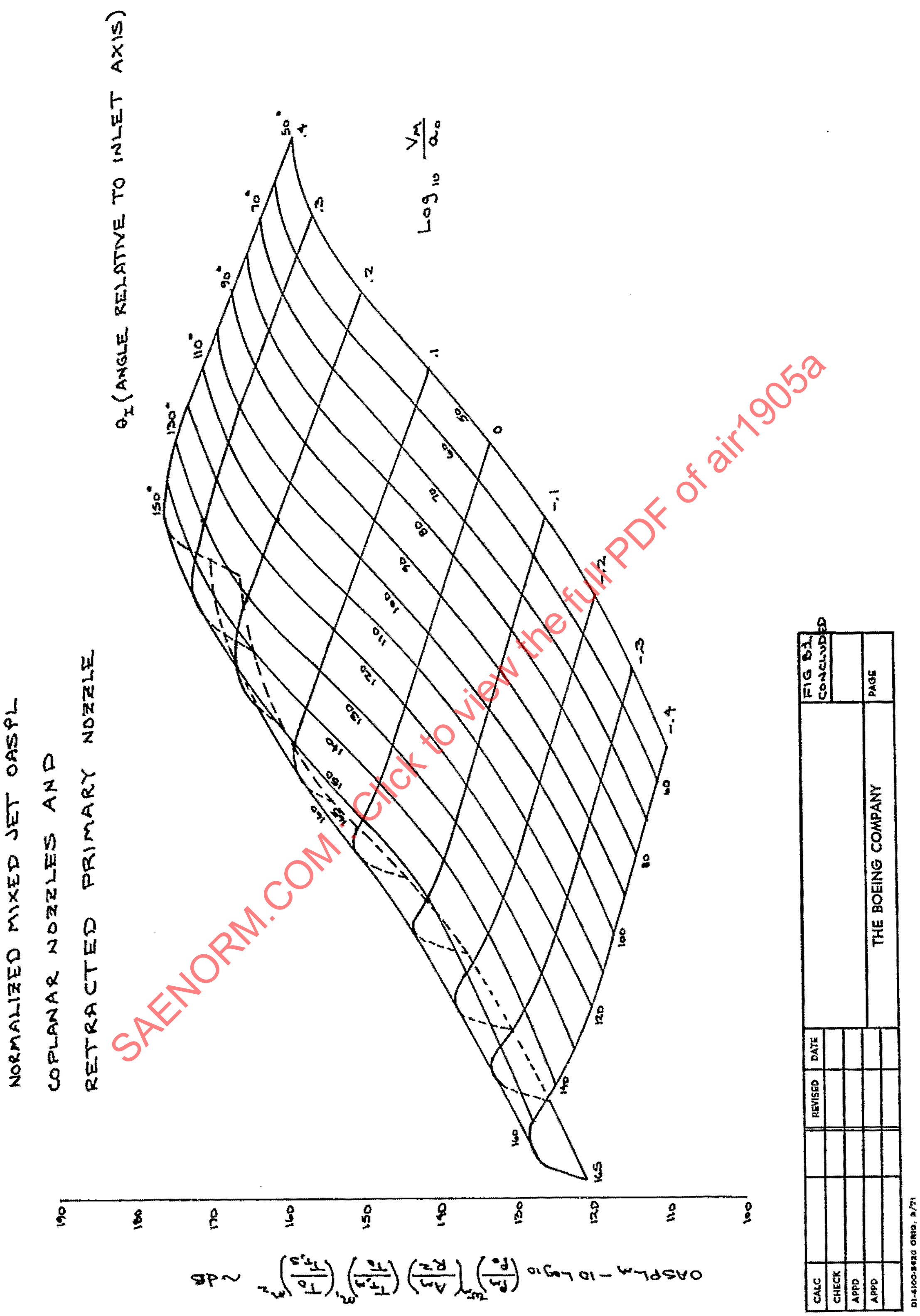

COAXIAL JET NOISE PREDICTION PROCEDURE
NORMALIZED MIXED JET OASPL
EXTENDED PRIMARY NOZZLE
θI (ANGLE RELATIVE TO INLET AXIS)
Log10 Vm/a0
m1
m2
θI, DEGREES
NORMALIZED MIXED JET OASPL
FIG. B1
THE BOEING COMPANY


NORMALIZED MIXED JET OASPL
COPLANAR NOZZLES AND
RETRACTED PRIMARY NOZZLE
θI (ANGLE RELATIVE TO INLET AXIS)
Log10 Vm/a0
OASPLm − 10 Log10 (ρ0/ρm)^ω3 (Am/R²) (Tt,m/Tt,s)^m1 (To/Tt,s)^m2 ~ dB
190
180
170
160
150
140
130
120
110
100
FIG B1 CONCLUDED
THE BOEING COMPANY
CALC
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APPD
APPD
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DATE
PAGE

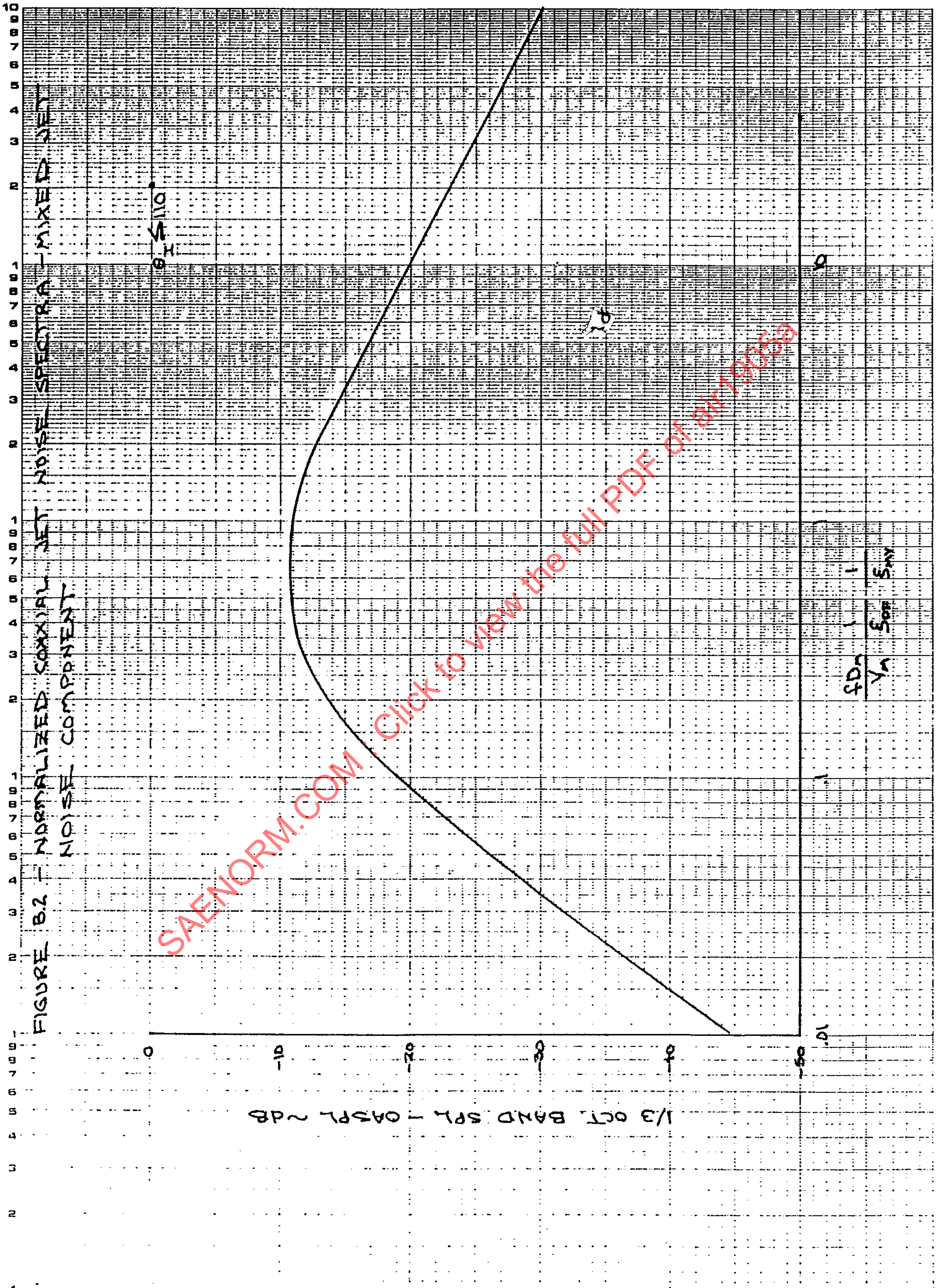


FIGURE B.2 – NORMALIZED COAXIAL JET NOISE SPECTRA – MIXED JET NOISE COMPONENT

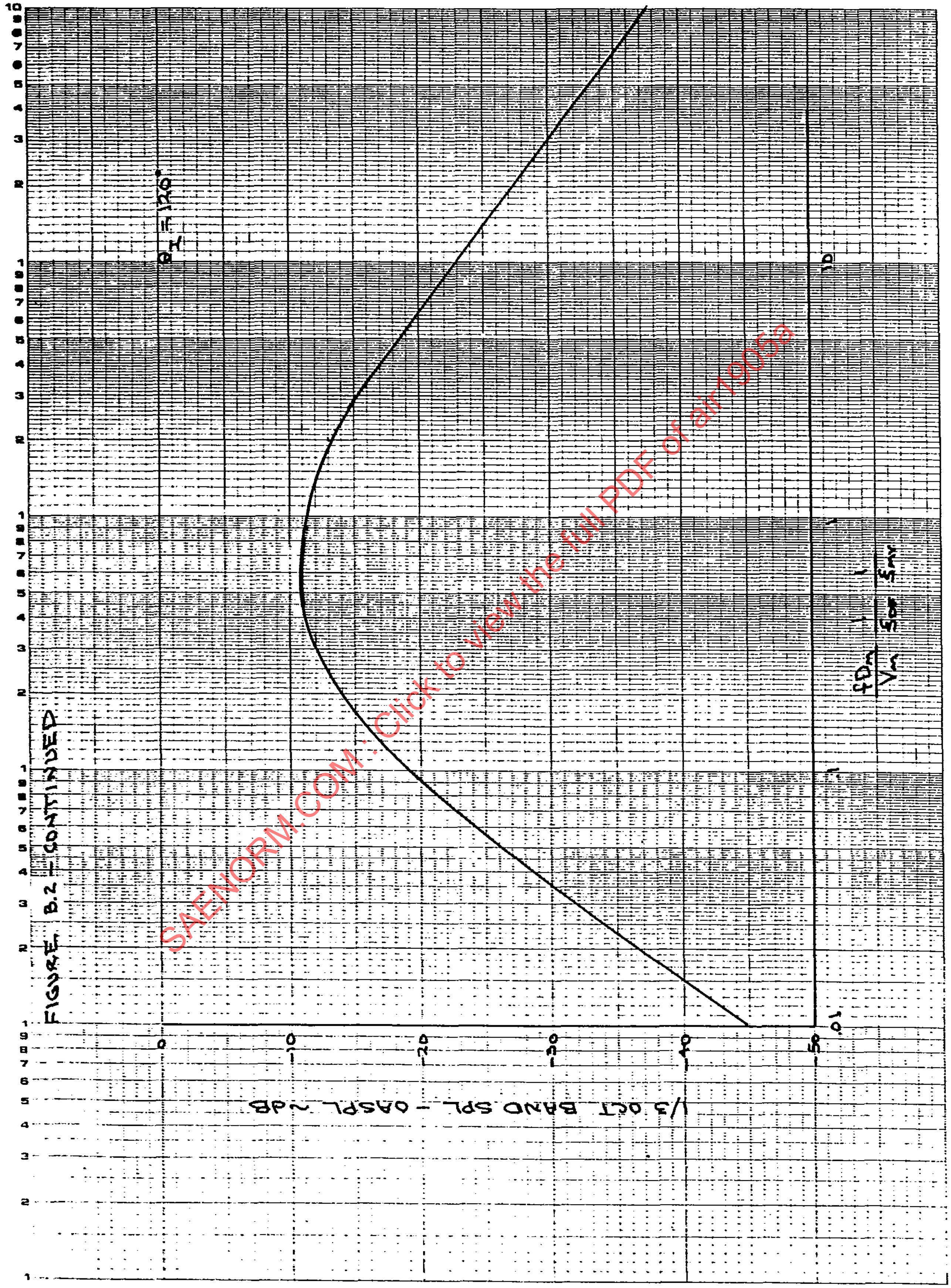


FIGURE B.2 – CONTINUED
θ_I = 120°
1/3 OCT BAND SPL – OASPL ~dB
0
-10
-20
-30
-40
-50
.01
.1
1
10
fD_m / V_m

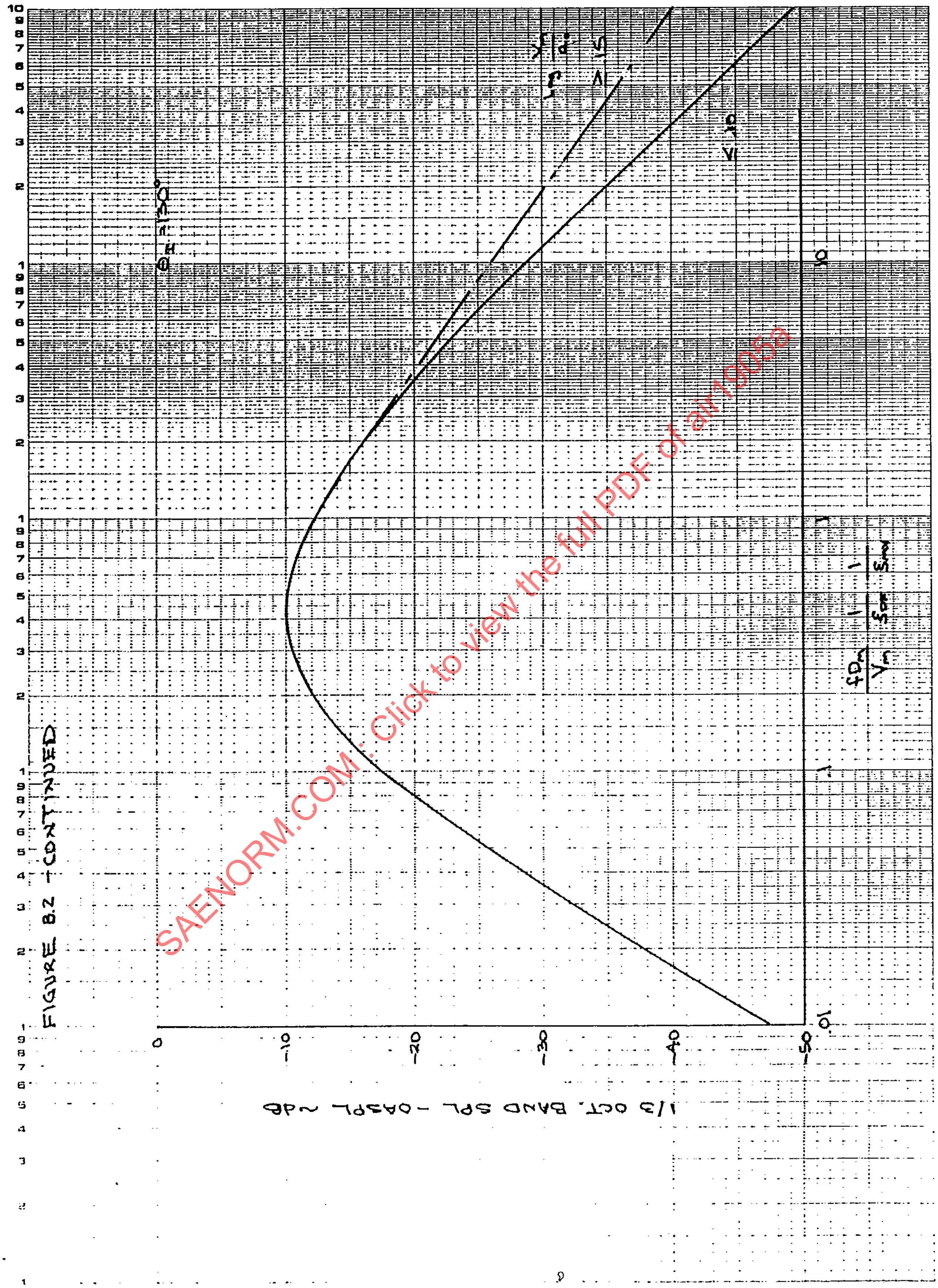


FIGURE B.2 - CONTINUED
θH = 130°
1/3 OCT. BAND SPL - OASPL ~dB
0
-10
-20
-30
-40
-50
.01
.1
1
10

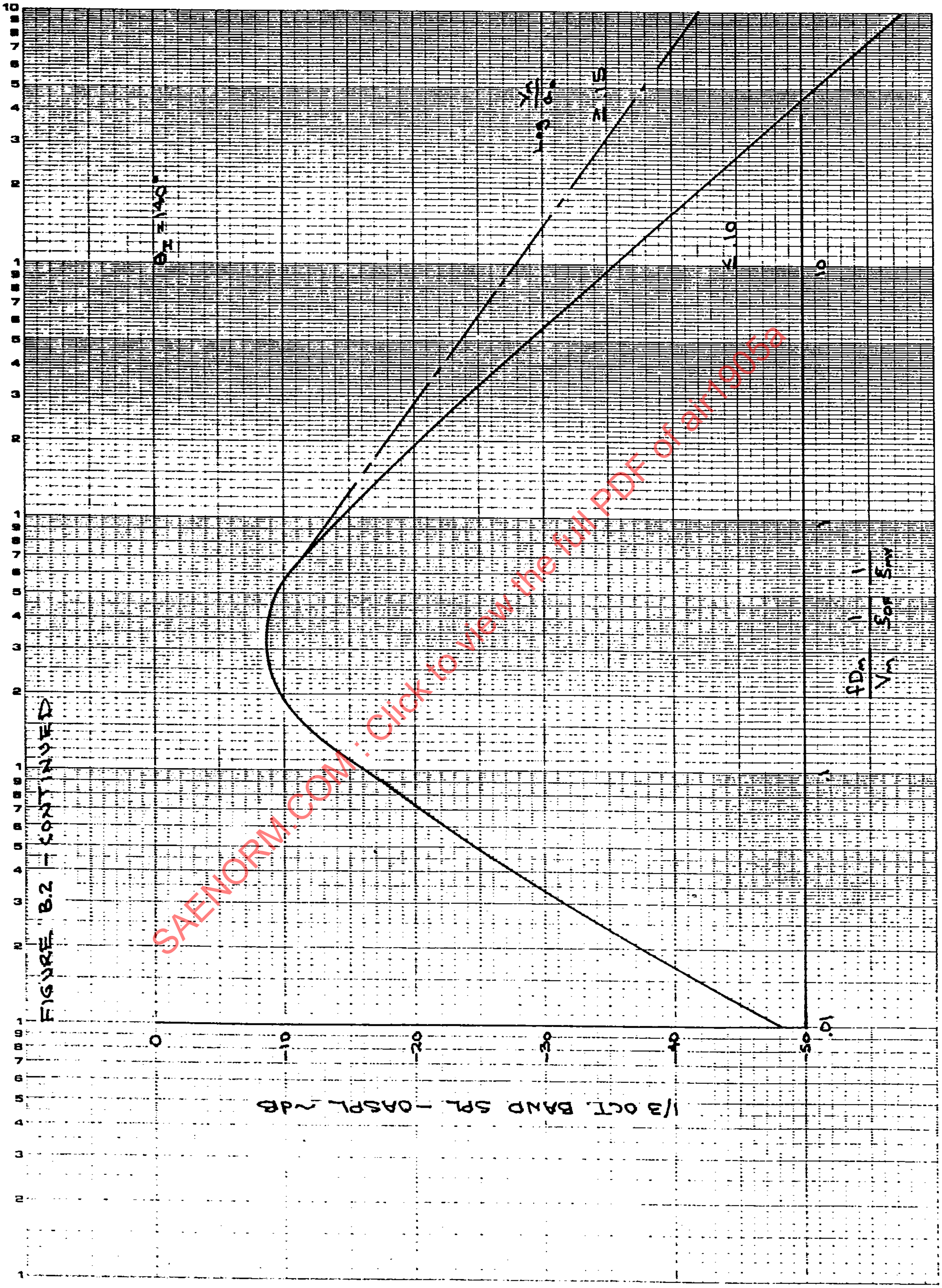


FIGURE B.2 - CONTINUED
1/3 OCT. BAND SPL - OASPL ~dB
0
-10
-20
-30
-40
-50

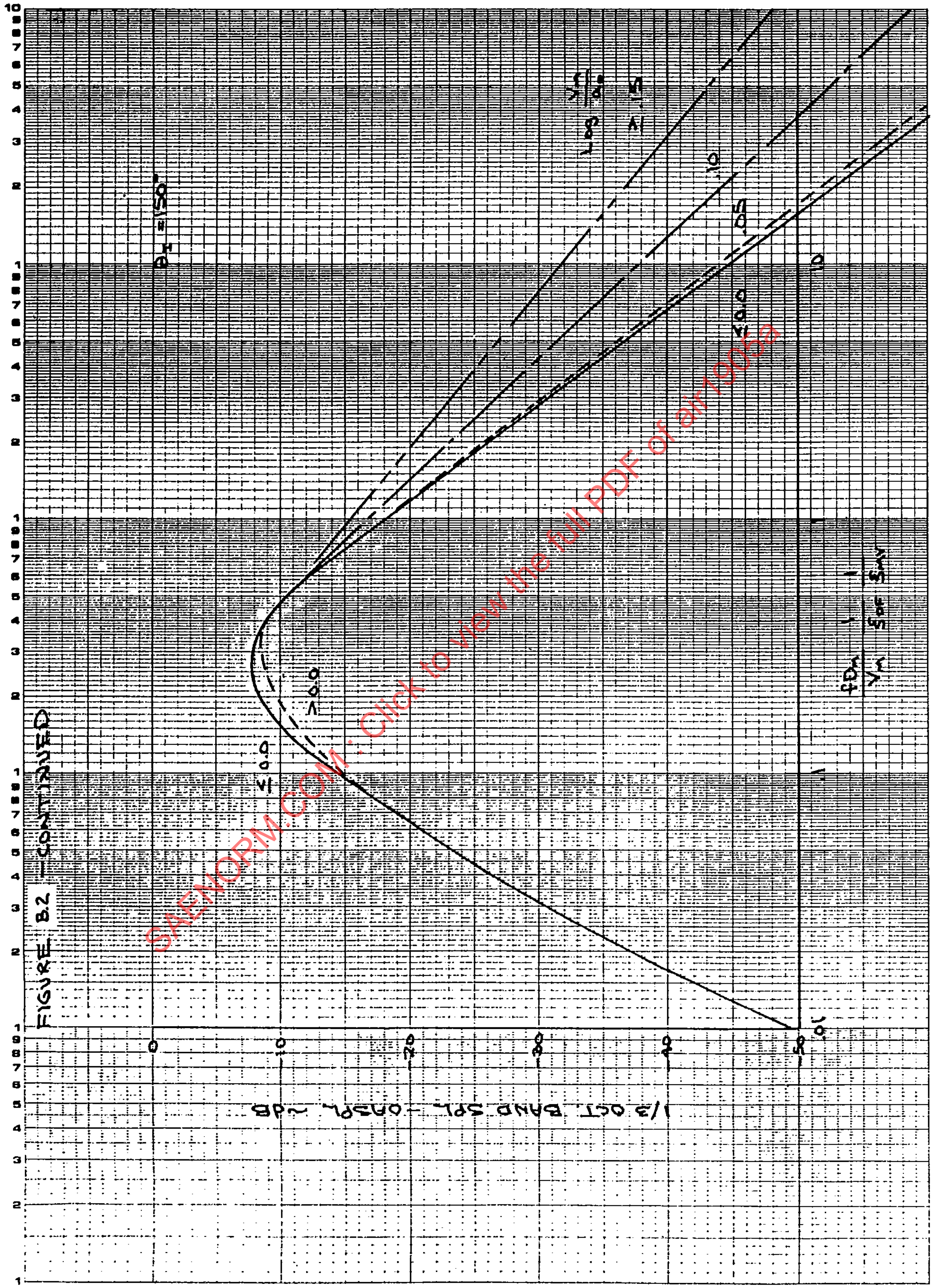


FIGURE B.2 —CONTINUED
1/3 OCT. BAND SPL - OASPL ~dB

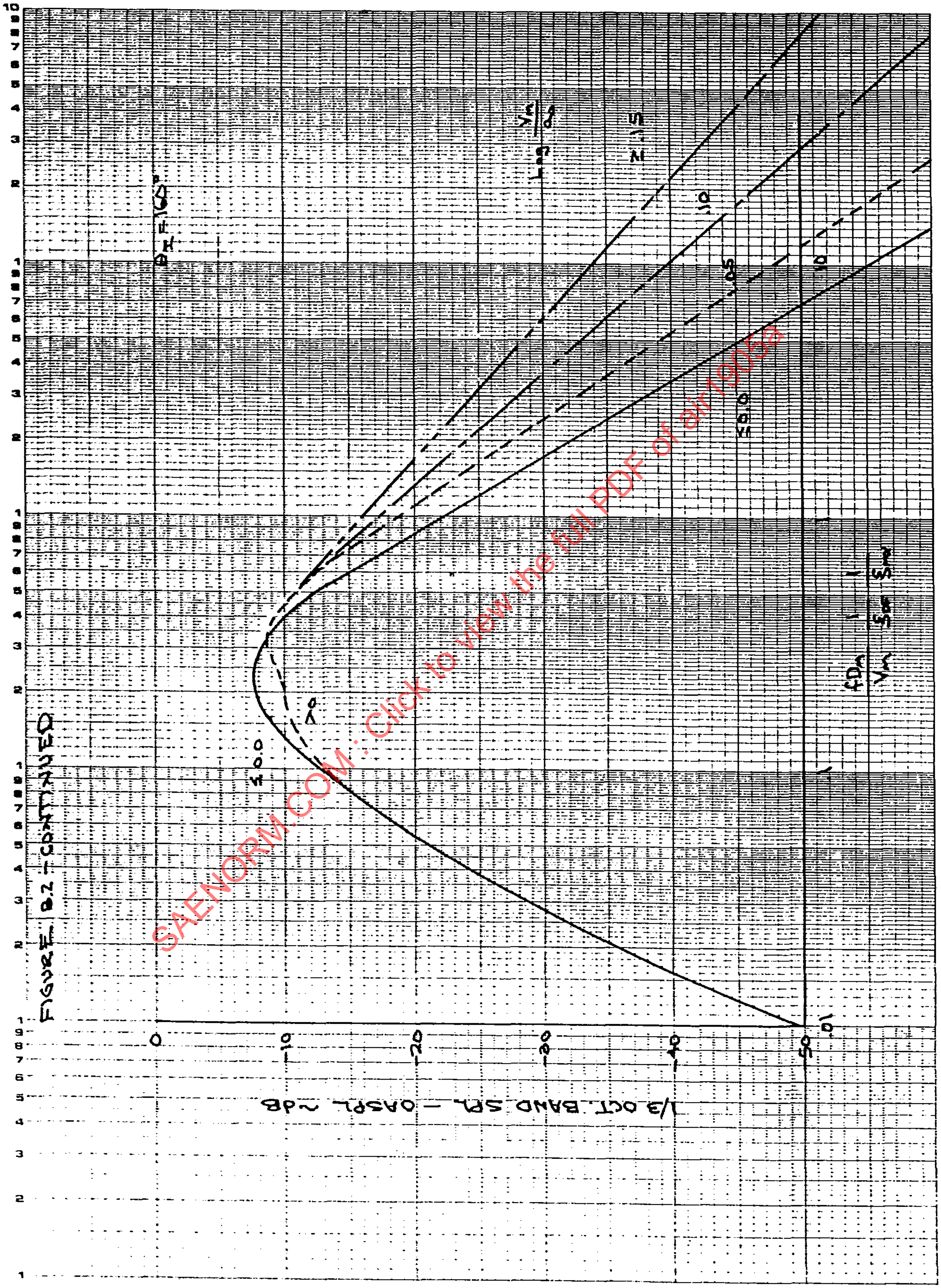


FIGURE B.2 - CONTINUED
1/3 OCT. BAND SPL - OASPL ~dB
0
-10
-20
-30
-40
-50
-10
$\frac{fD_m}{V_m}\frac{1}{S_{or}}\frac{1}{S_m}$
2.15
1.0
.05
10
0.05
0.02

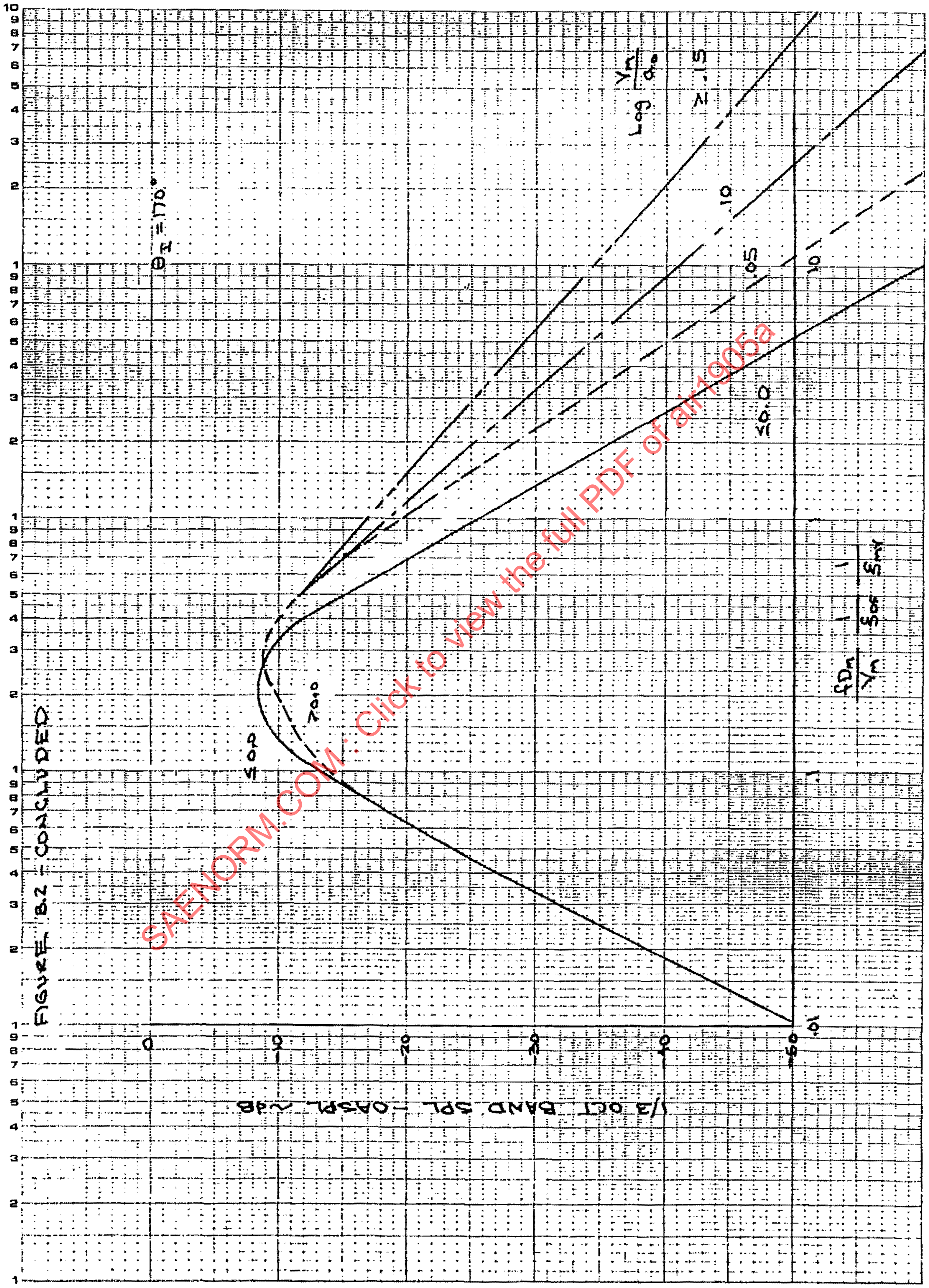
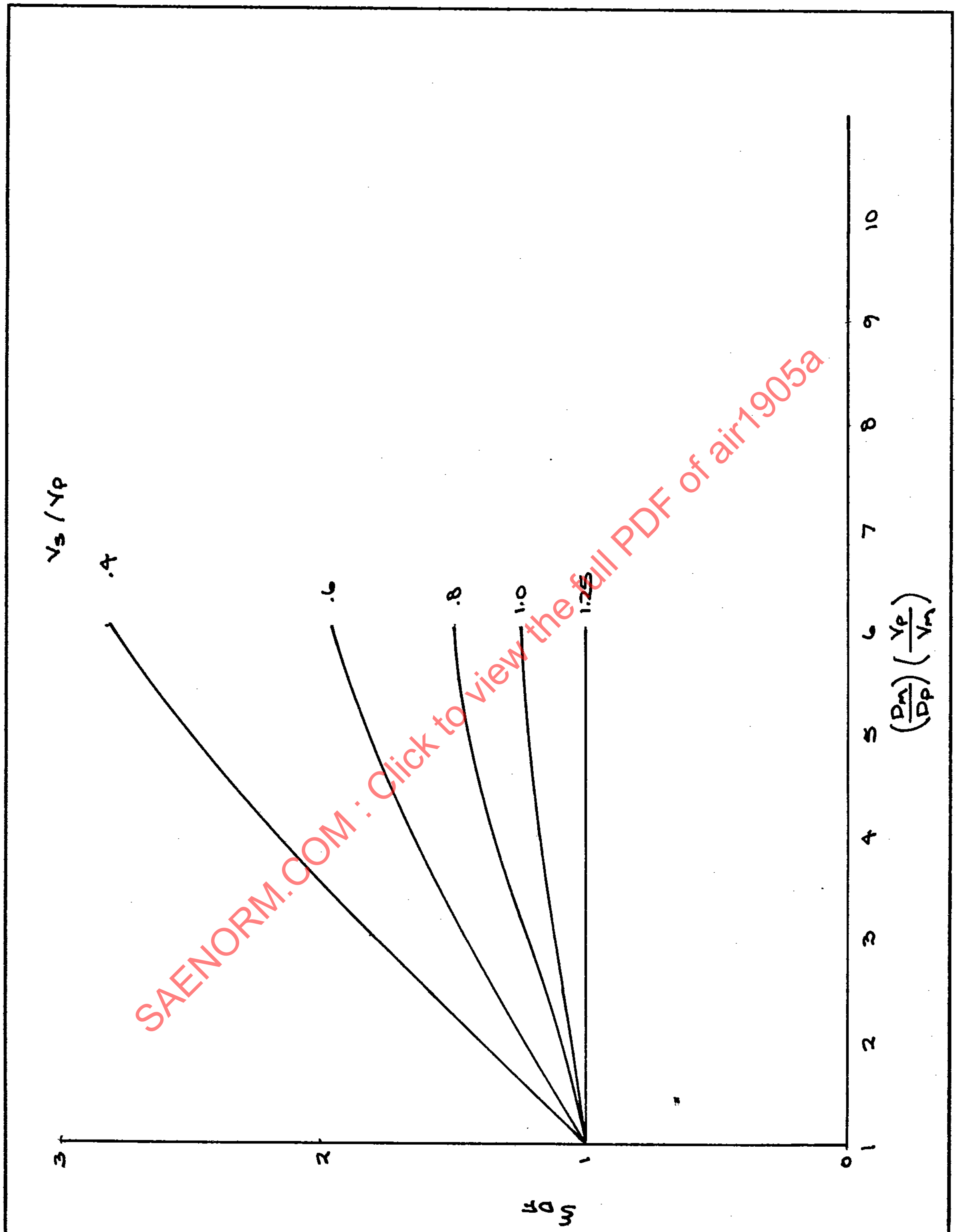


FIGURE B.2 - CONCLUDED
θI = 170°
Log Vn/a0
≥ .15
.10
.05
≤0.0
fDn/Vn
1/3 OCT BAND SPL - OASPL ~ dB
0
-10
-20
-30
-40
-50



CALC			REVISED	DATE	COAXIAL JET STROUHAL CORRECTION FACTOR	FIG. B-3
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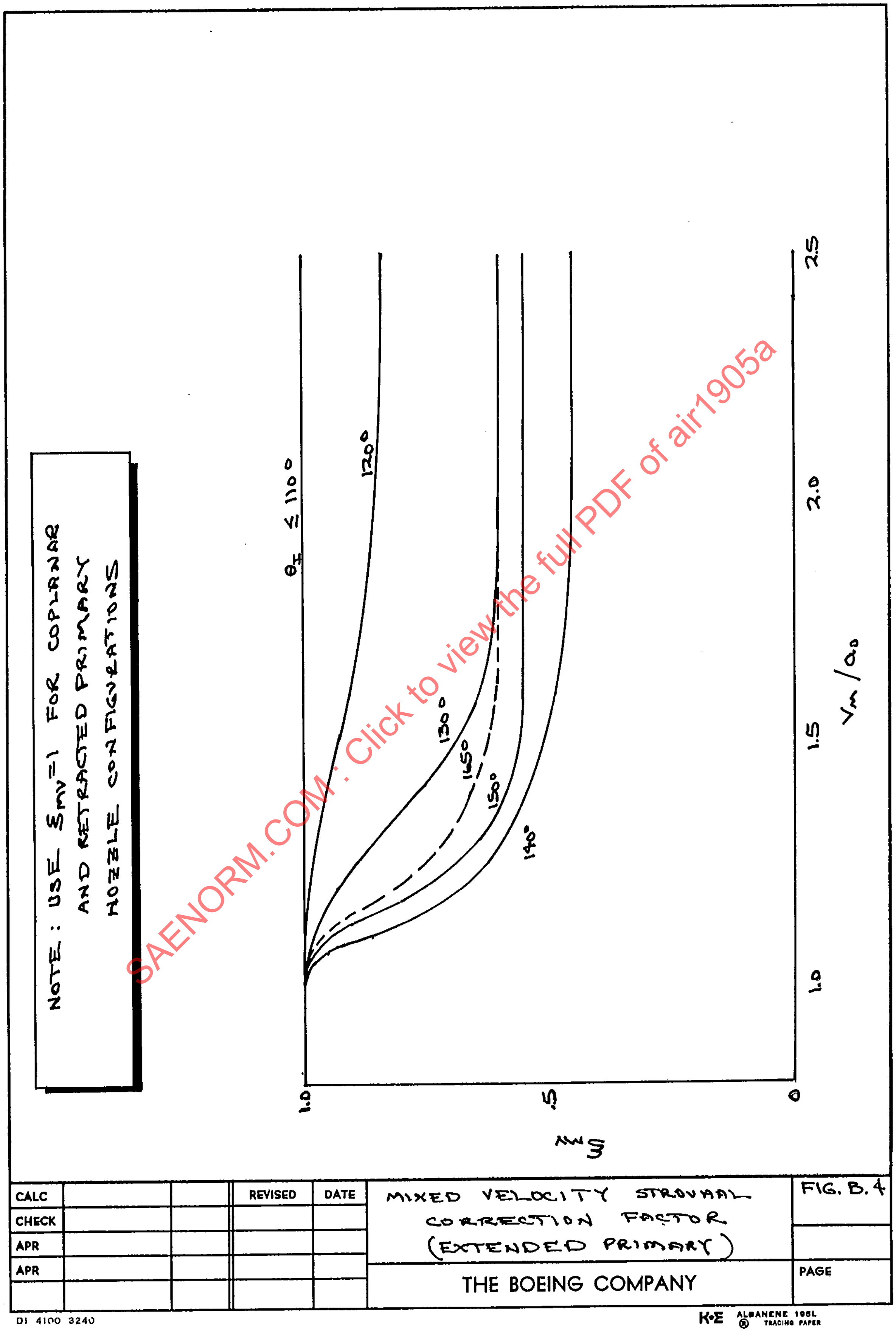
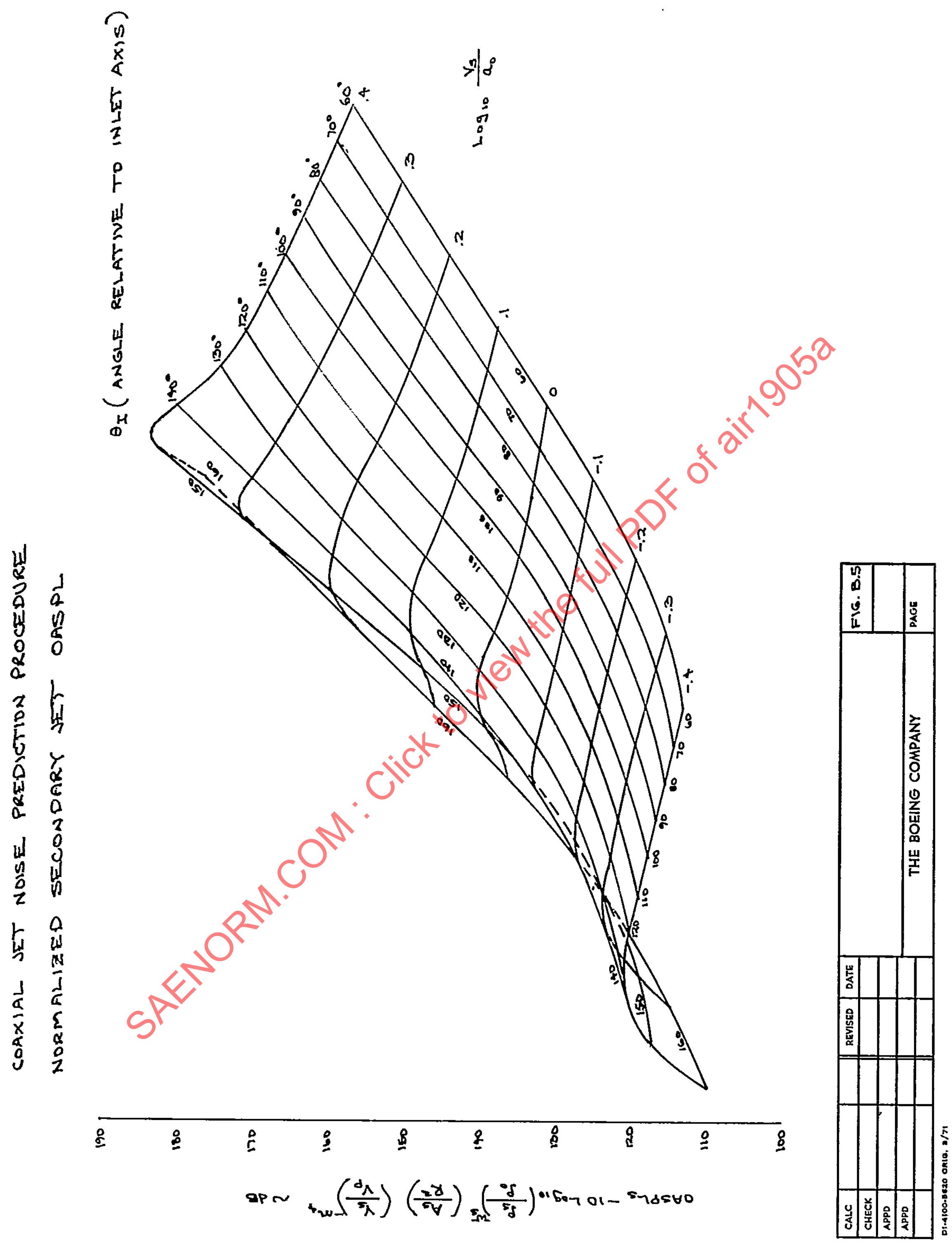
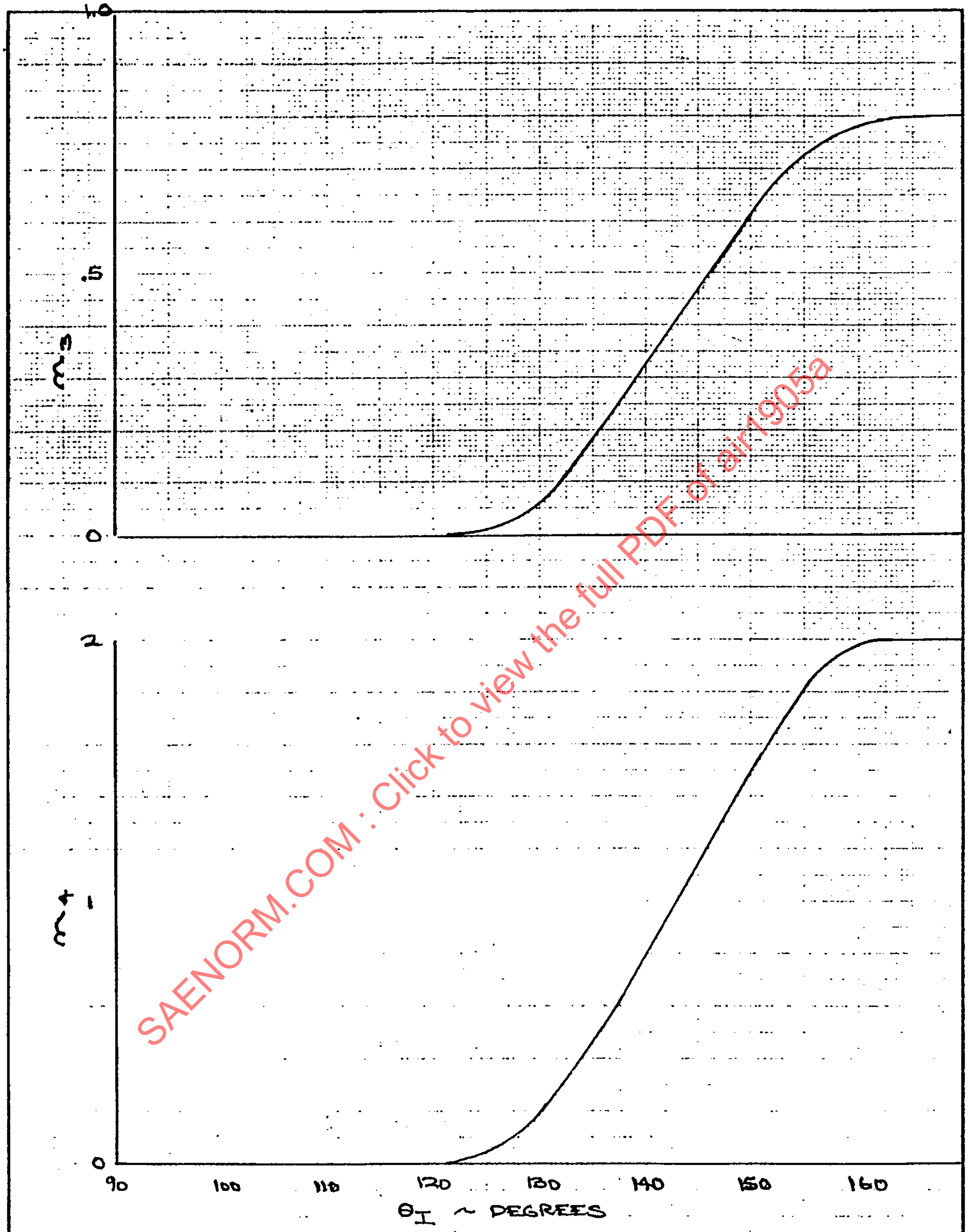
APR					THE BOEING COMPANY	PAGE

D1 4100 3240

K+E ALBANENE 195L TRACING PAPER

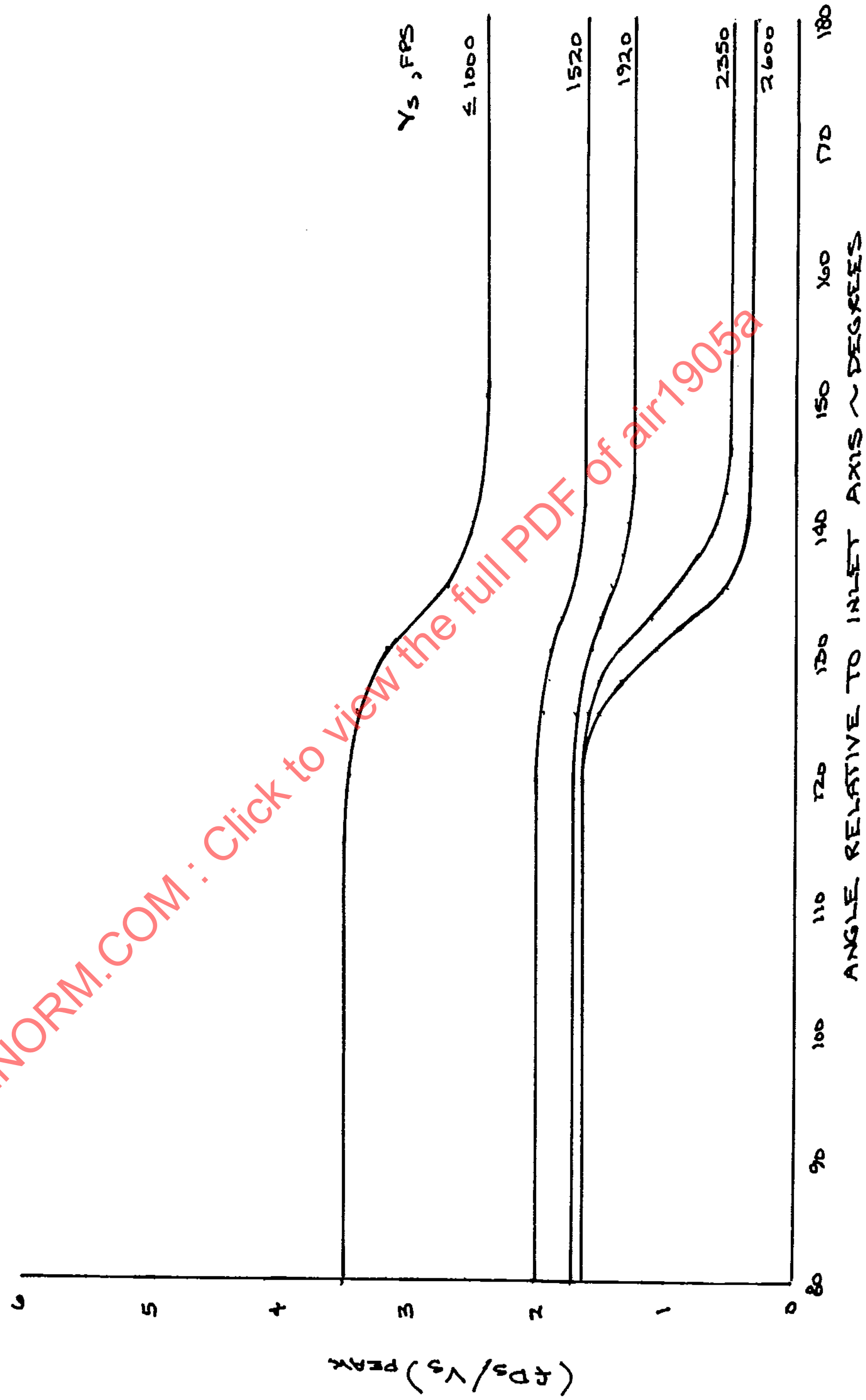

NOTE: USE $\varepsilon_{MV}=1$ FOR COPLANAR AND RETRACTED PRIMARY NOZZLE CONFIGURATIONS
$\theta_j \le 110°$
120°
130°
165°
150°
140°
1.0
.5
0
ε_{MV}
1.0
1.5
2.0
2.5
V_m/a_0
CALC
CHECK
APR
APR
REVISED
DATE
MIXED VELOCITY STROUHAL CORRECTION FACTOR (EXTENDED PRIMARY)
THE BOEING COMPANY
FIG. B.4
PAGE
DI 4100 3240
K+E ALBANENE 195L TRACING PAPER


COAXIAL JET NOISE PREDICTION PROCEDURE
NORMALIZED SECONDARY JET OASPL
FIG. B.5
THE BOEING COMPANY



CALC			REVISED	DATE	EXPONENTS FOR SECONDARY JET VELOCITY RATIO CORRECTIONS TO OASPL AND STROUHAL NUMBER	FIG. B.6
CHECK						
APR						
APR					THE BOEING COMPANY	PAGE

D1 4100 3240 K•E ALBANENE 195L



CALC			REVISED	DATE	SECONDARY JET PEAK STROUHAL NUMBERS	FIG. B.7
CHECK						
APR					THE BOEING COMPANY	PAGE
APR						

K+E ALBANENE 195L TRACING PAPER

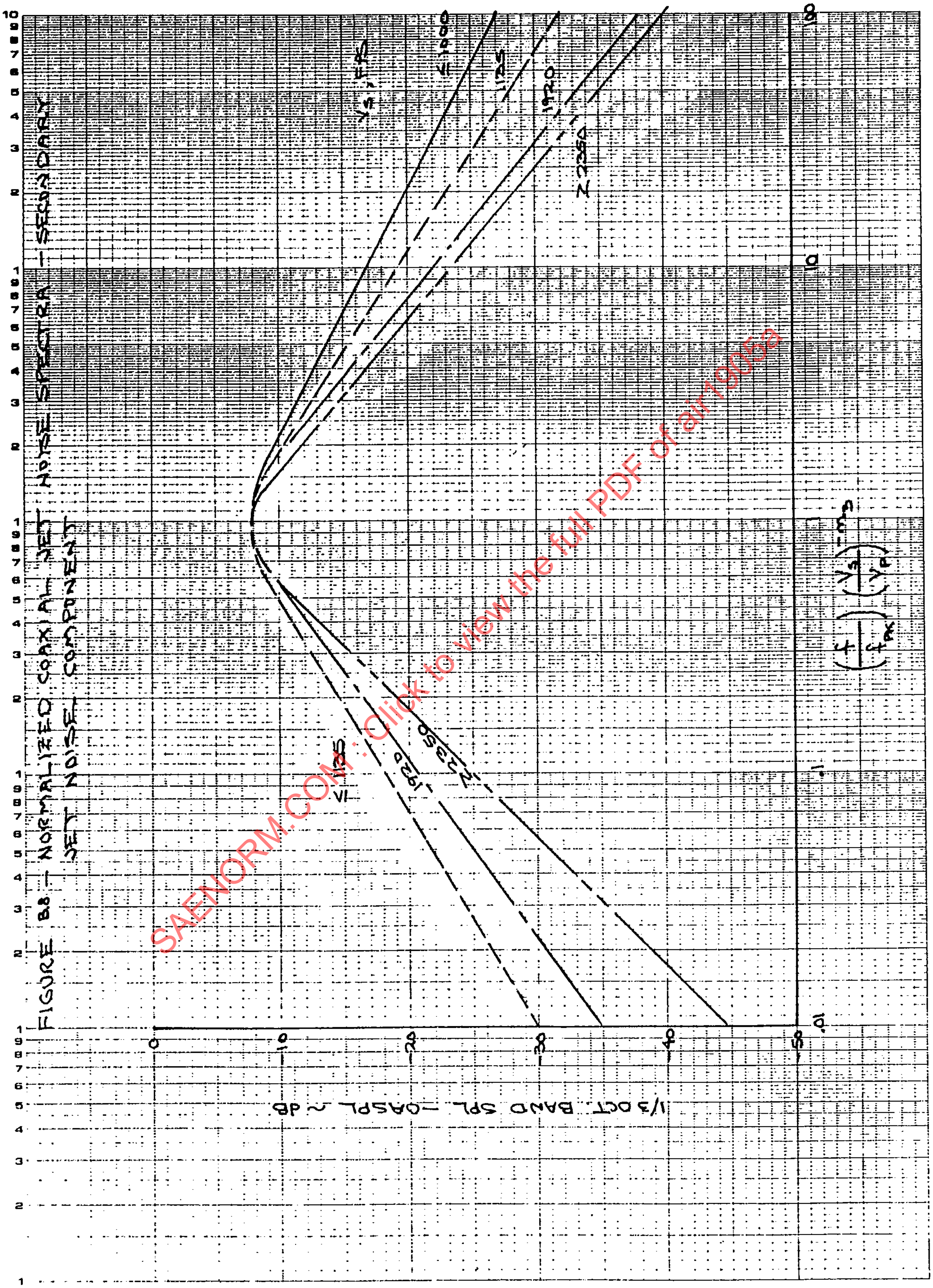


FIGURE B.8 - NORMALIZED COAXIAL JET NOISE SPECTRA - SECONDARY JET NOISE COMPONENT

METHOD 3

(ex-NASA Langley)

7. PREDICTION OF SINGLE STREAM AND DOUBLE STREAM STATIC JET MIXING NOISE FROM SHOCK FREE CIRCULAR AND COAXIAL NOZZLES:

7.1 Static Conditions: An extensive jet noise data base has been developed from nine separate test series of model data consisting of 214 different circular jet test points and 603 coaxial jet test points from five different industry and government sources. Analysis of the data shows free field jet mixing noise from subsonic circular jets can be defined from the jet velocity and jet total temperature. For coaxial jets the free field jet mixing noise with both jets subsonic requires three additional parameters, the jet velocity ratio, the jet total temperature ratio, and the jet area ratio. The prediction parameters and the recommended range of operation for which the method is valid are listed below.

Prediction Parameter	Operating Range
a) Normalized equivalent jet velocity, V_e/a_o	0.3 to 2.0
b) Normalized equivalent jet total temperature, T_e/t_o	0.7 to 4.5
c) Ratio of secondary jet velocity to primary jet velocity, V_2/V_1	0.02 to 2.5
d) Ratio of secondary jet total temperature to primary jet total temperature, T_2/T_1	0.2 to 4.0
e) Ratio of secondary jet area to primary jet area, A_2/A_1	0.5 to 10.0

The free field circular or coaxial jet mixing noise one-third octave band sound pressure level can be expressed in components as:

$$SPL(\theta_i, \eta) = \overline{OAPWL} + DI(\theta_i) + F(\eta) + RSL(\theta, \eta) + 20\log_{10}\left(\frac{\rho_o a_o^{\,2}}{\rho_{ISA} a_{ISA}^{\,2}}\right) + 10\log_{10}\left(\frac{A_{ref}}{4\pi r^2}\right) + 197.0 \quad (1)$$

7.1 (Continued):

where $\overline{OAPWL}$ is the normalized overall power level defined by

$$\overline{OAPWL} = 10 \log_{10} \quad [\text{Acoustic Power}/(\dot{m}_1 + \dot{m}_2)\ a_o^2)] \tag{2}$$

and the reference area, A_{ref}, is the area of a cold jet at the critical pressure ratio which has the same mass flow as the hot jet and is defined as

$$A_{ref} = (\dot{m}_1 + \dot{m}_2)/(\rho_o a_o) \tag{3}$$

and r is the distance from the nozzle exit centerline to the observer.

The directivity angle, θ_i, is the angle relative to the inlet axis and the parameter η represents the logarithm of the one-third octave band normalized frequency. The normalized frequency parameter, η, is related to the frequency, f, by

$$\eta = \log_{10} (f\ D_e/V_e) \tag{4}$$

where V_e and D_e are the equivalent jet velocity and jet diameter for a circular or coannular jet.

$DI(\theta_i)$ is the normalized directivity, $F(\eta)$ is the normalized power spectrum, and RSL (θ_i, η) is the normalized relative spectrum. The overall sound pressure level, OASPL, and the one-third octave band power spectrum level, PWL, can be expressed as

$$OASPL(\theta_i) = \overline{OAPWL} + DI(\theta_i) + 20 \log_{10} \left(\frac{\rho_o a_o^2}{\theta_{ISA} a_{ISA}^2}\right) + 10 \log_{10} \left(\frac{A_{ref}}{4\pi r^2}\right) + 197.0 \tag{5}$$

$$PWL(\eta) = \overline{OAPWL} + F(\eta) + 20 \log_{10} \left(\frac{\rho_o a_o^2}{\rho_{ISA} a_{ISA}^2}\right) + 10 \log_{10} \quad ((\dot{m}_1 + \dot{m}_2) a_o^2/\pi_{ref}) + 197.0 \tag{6}$$

Once the values $\overline{OAPWL}$, $F(\eta)$, $D(\theta_i)$, and RSL (θ_i,η) are defined any of the desired noise level quantities can be computed from equations 1, 5, or 6.

7.1 (Continued):

The method of calculation is as follows:

Step 1 - Calculate the equivalent flow properties for the coaxial jet. The single equivalent jet has the same mass flow, energy flow, and thrust as the coaxial jet. The mass flow of the single equivalent jet is:

$$\dot{m}_e = \dot{m}_1 + \dot{m}_2 \quad \text{where} \quad \dot{m} = \rho AV . \tag{7}$$

The condition of equivalence of mass flow and thrust gives the equivalent velocity, V_e, as

$$V_e = \frac{\dot{m}_1 V_1 + \dot{m}_2 V_2}{\dot{m}_1 + \dot{m}_2} \tag{8}$$

Since the gas constant, R, for air is not significantly changed by the addition of a small amount of combustion products, the equivalent temperature can be defined from the total energy flow as

$$T_e = \frac{\dot{m}_1 \frac{\gamma_1}{(\gamma_1 - 1)} T_1 + \dot{m}_2 \frac{\gamma_2}{(\gamma_2 - 1)} T_2}{\dot{m}_1 \frac{\gamma_1}{(\gamma_1 - 1)} + \dot{m}_2 \frac{\gamma_2}{(\gamma_2 - 1)}} \tag{9}$$

where the specific heat ratio, γ, is defined as, $\frac{\gamma}{\gamma-1} = \frac{c_p}{R}$, where c_p is the specific heat at constant pressure.

The equivalent specific heat ratio, γ_e, is defined from the mixing of the gases of each stream by

$$\frac{\gamma_e}{(\gamma \;\; - 1)} = \frac{\dot{m}_1 \frac{\gamma_1}{(\gamma_1 - 1)} + \dot{m}_2 \frac{\gamma_2}{(\gamma_2 - 1)}}{\dot{m}_1 + \dot{m}_2} \tag{10}$$

Because the fully expanded jet static pressure is equal to the ambient pressure the equivalent jet density, ρ_e, can be defined from the ambient jet density, ρ_o, by

$$\rho_e = \rho_o\left(\frac{T_e}{t_o} - \frac{\gamma_e - 1}{2}\left(\frac{V_e}{a_o}\right)^2\right)^{-1} \tag{11}$$

The equivalent jet area, A_e, is then defined from continuity as

$$A_e = \frac{\dot{m}_e}{\rho_e V_e} \tag{12}$$

and the equivalent jet diameter, D_e, is

$$D_e = (4\ A_e/\pi)^{1/2} \tag{13}$$

Also the reference area, A_{ref}, needs to be computed using equation 3 as follows

$$A_{ref} = \dot{m}_e/(\rho_o a_o) \tag{3}$$

Step 2 - Calculate the parameters x_1 to x_5 from the equivalent jet flow properties and the coaxial jet flow property ratios as follows

$$x_1 = \log_{10}(V_e/a_o) \tag{14}$$

$$x_2 = \log_{10}(T_e/(2\ t_o)) \tag{15}$$

$$x_3 = \log_{10}(V_2/V_1) \tag{16}$$

$$x_4 = \log_{10}(T_2/T_1) \tag{17}$$

$$x_5 = \log_{10}(A_2/A_1) \tag{18}$$

For the circular jet $V_e = V_1$, $T_e - T_1$. and x_3, x_4, and x_5 are equal to 0.0.

Step 3 - Using the values x_1 to x_5 obtained in Step 2 compute the values of the derivative multipliers, X_1 to X_N, as listed in Table C1, where N has a value of 8 for the circular jet and a value of 36 for the coannular jet.

7.1 (Continued):

Step 4 - Compute the value of the normalized overall power level, $\overline{OAPWL}$, from the derivative multiplier values, X_j, and the corresponding N number of derivative values, $PWL,_j$, listed in Table C2 where N is equal to 8 for the circular jet and 36 for the coannular jet using the relation

$$\overline{OAPWL} = \sum_{j=1}^{N} PWL,_j \, X_j \tag{19}$$

Step 5 - Compute the values of the directivity index, $DI(\theta_i)$ for θ_i values of 0°, 30°, 60°, 90°, 120°, 150°, and 180°, from the derivative multiplier values, X_j, and the corresponding number N of directivity index derivative values, $DI,_j(\theta_i)$, listed in Table C2 using the relation

$$DI(\theta_i) = \sum_{j=1}^{N} DI,_j(\theta_i) \, X_j, \tag{20}$$

for θ_i = 0°, 30°, 60°, 90°, 120°, 150°, and 180°.

Step 6 - Compute the values of the normalized power spectrum, $F(\eta)$, for η values of -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, and 1.5 from the derivative multiplier values, X_j, and the corresponding number N of power spectrum derivative values, $F,_j(\eta)$ listed in Table C3 using the relation

$$F(\eta) = \sum_{j=1}^{N} F,_j(\eta) \, X_j \tag{21}$$

for η = -1.5, -1.0, -0.5, 0.0, 0.5, .0, and 1.5.

Step 7 - Compute the values of the normalized relative spectrum, RSL (θ_i,η) for θ_i values of 0°, 30°, 60°, 90°, 120°, 150°, and 180° and for η values of -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, and 1.5 from the derivative multiplier values, X_j, and the corresponding derivative values, $RSL,_j(\theta_i,\eta)$, in Table C4 using the relation

$$RSL\,(\theta_i,\eta) = \sum_{j=1}^{N} RSL,_j\,(\theta_i,\eta) \, X_j \tag{22}$$

for θ_i = 0°, 30°, 60°, 90°, 120°, 150°, and 180°

and for η = 1.5, -1.0, -0.5, 0.0, 0.5, 1.0, and 1.5.

7.1 (Continued):

Step 8 - Compute the values of OASPL (θ_i) for θ_i values of 0°, 30°, 60°, 90°, 120°, 150°, and 180°, from the value of $\overline{OAPWL}$ obtained in Step 4, the values of DI(θ_i) obtained in Step 5 and the value of A_{ref} obtained in Step 1 using equation 5 shown below

$$OASPL(\theta_i) = \overline{OAPWL} + DI(\theta_i) + 20 \log_{10} \left(\frac{\rho_o a_o^2}{\rho_{ISA} a_{ISA}^2}\right) + 10 \log_{10} \left(\frac{A_{ref}}{4\pi r^2}\right) + 197.0 \qquad (23)$$

Step 9 - Compute the OASPL values at the desired directivity angles (θ_D) by interpolating the OASPL (θ_i) values obtained in Step 8 to obtain the OASPL (θ_D) values using a cubic spline which has zero slope end conditions (directivity angles of 0° and 180°). The cubic spline is a piecewise third order polynomial with continuous slope and curvature at the node points θ_i.

Step 10 - Compute the relative spectrum level values at the desired directivity angles by interpolating the RSL (θ_i,η) values obtained in Step 7 using a cubic spline which has zero slope end conditions for normalized frequency parameter (η) values of -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, and 1.5 to obtain the RSL(θ_D,η) values.

Step 11 - For the desired one third octave band frequencies, (f_D) compute the value of the parameter, η, to obtain the desired η_D values using equation 4 as shown

$$\eta_D = \log_{10} (f_D D_e/V_e) \qquad (24)$$

where f_D is the desired one third octave band center frequency and V_e and D_e are the equivalent velocity and equivalent diameter computed in Step 1.

Step 12 - Compute the normalized power spectrum levels and the relative spectrum level values at the desired η_D values obtained in Step 11 and the desired directivity angles by interpolating the F(η) values obtained in Step 6 and the RSL (θ_D,η) values obtained in Step 10 to obtain the values (F(η_D) and the values of RSL (θ_D, η_D) using a cubic spline with zero curvature end conditions (η = -1.5 and +1.5).

7.1 (Continued):

Step 13 - Compute the sound pressure level values at the desired directivity angles and desired frequency levels from the values of $F(\eta_D)$ and $RSL(\theta_D, \eta_D)$ obtained in Step 12 and the OASPL (θ_D) values obtained in Step 9 to obtain the values for SPL (θ_D, η_D) using the relation

$$SPL(\theta_D,\eta_D) = OASPL(\theta_D) + F(\eta_D) + RSL(\theta_D,\eta_D) \tag{25}$$

Step 14 - Also the one third octave band power spectrum level at the desired frequencies can be computed from the $\overline{OAPWL}$ value obtained in Step 4 and the $F(\eta_D)$ values obtained in Step 12 to obtain the values for $PWL(\eta_D)$ using the relation

$$PWL(\eta_D) = \overline{OAPWL} + F(\eta_D) + 20 \log_{10}\left(\frac{\rho_o a_o^2}{\rho_{ISA} a_{ISA}^2}\right) + 10 \log_{10} (\dot{m}_e a_o^2/\pi_{ref}) + 197.0 \tag{26}$$

Note 1 - The power spectrum levels and relative spectrum levels generated in Steps 6 and 7 are normalized such that

$$\sum_n 10^{\left(\frac{F(\eta)}{10}\right)} = 1$$

and

$$\sum_n 10^{\left(\frac{RSL(\theta_i,\eta)}{10}\right)} = 1 \quad \text{for each } \theta_i$$

Note 2 - The overall directivity index generated in Step 4 is normalized such that

$$\frac{1}{2} \int_0^{\pi} 10^{\left(\frac{DI(\theta_i)}{10}\right)} \sin \theta_i \, d\theta_i = 1$$

7.1 (Continued):

Note 3 - The standard error of the prediction method is dependent on the availability of the data. At directivity angles of 0° and 180° where there are no actual data the standard error of extrapolated values is generally greater than 2 dB. At η values of +1.5 and -1.5 where there are less data the standard error is greater than at the mid range of the η values. The standard error values over the directivity angle (θ_i) range of 0° to 180° and the frequency parameter (η) range from -1.5 to 1.5 are listed in Table C5. Figure C1 shows a contour plot of the standard error values. The mean error is 0.0 dB everywhere.

Note 4 - The symbols presented in Section 4 apply to this Appendix, however for completeness all the symbols used in this Appendix are presented below:

SYMBOLS

a_o	Ambient speed of sound	m/s
a_{ISA}	Speed of sound under ISA, SL conditions	m/s
A_e	Nozzle exit area of equivalent single jet	m^2
A_{ref}	Reference area used in computing normalized power	m^2
A_1	Nozzle exit area of inner stream of circular jet	m^2
A_2	Nozzle exit area of outer stream	m^2
c_p	Specific heat value at constant pressure	J/kg k
$DI(\theta_i)$	Directivity Index	dB
$DI,_j(\theta_i)$	Derivative value for Directivity Index	
D_e	Nozzle exit diameter of equivalent single jet	m
f	One-third octave band center frequencies	Hz
f_D	Desired one-third octave band center frequencies at which noise levels are computed	Hz

7.1 (Continued):

F	One-third octave band normalized power spectrum	dB
$F_{,j}(\eta)$	Derivative value for one-third octave band normalized power spectrum	
ISA	International standard atmosphere	
j	Index for derivative value	
$\dot{m}_e$	Mass flow rate of single equivalent jet	kg/s
$\dot{m}_1$	Mass flow rate of inner stream or circular jet	kg/s
$\dot{m}_2$	Mass flow rate of outer stream	kg/s
N	Number of derivatives and derivative multipliers employed in computation of OAPWL, $DI(\theta_i)$, $F(\eta)$ and $RSL(\theta_i,\eta)$	
$\overline{OAPWL}$	Normalized overall acoustic power level	dB
$OASPL\ (\theta_i)$	Overall sound pressure level	dB
$PWL(\eta)$	1/3 octave band power spectrum level	dB
$PWL_{,j}$	Derivative values for overall acoustic power level	
r	Radial distance from nozzle exit to observer	m
R	Gas constant for air	J/kg k
$RSL(\theta_i,\eta)$	Normalized relative spectrum level	dB
$RSL_{,j}(\theta_i,\eta)$	Derivative values of normalized relative spectrum level	
SPL	Sound pressure level	dB
t_o	Ambient static temperature	k
T_e	Nozzle exit equivalent jet total temperature	k

7.1 (Continued):

T_1	Nozzle exit total temperature of inner stream or circular jet	k
T_2	Nozzle exit total temperature of outer stream	k
V	Nozzle exit flow velocity	m/s
V_e	Nozzle exit equivalent jet flow velocity	m/s
V_1	Nozzle exit jet flow velocity of inner stream or circular jet	m/s
V_2	Nozzle exit jet flow velocity of outer stream	m/s
x	Nozzle exit flow parameters	
X_j	Derivative multiplier values	

GREEK SYMBOLS

γ	Ratio of specific heats	
γ_e	Nozzle exit flow specific heat ratio of equivalent jet	
γ_1	Nozzle exit flow specific heat ratio of inner stream or circular jet	
γ_2	Nozzle exit flow specific heat ratio of outer stream	
η	Normalized frequency parameter	
η_D	Desired values of frequency parameter at which noise levels are computed	
θ_i	Directivity angle relative to inlet axis	degrees
θ_D	Desired values of directivity angle for which noise levels are computed	degrees
π_{ref}	Acoustic reference power	watt
ρ	Density	kg/m^3
ρ_e	Nozzle exit flow density of equivalent jet	kg/m^3
ρ_{ISA}	Air density under ISA, SL conditions	kg/m^3
ρ_0	Ambient air density	kg/m^3

7.2 Parties Contributing to Formulation of Appendix C

Lockheed Georgia Company, USA

National Aeronautics and Space Administration, USA

Royal Aircraft Establishment, England

Pratt & Whitney Aircraft, USA

SNECMA, France